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**SECURITY DESIGN IN MARKETS WITH RISK: PRICE AND ALLOCATION
EFFICIENCIES**

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ABSTRACT

This research utilizes an experimental approach to examine the impact that security design has on market outcomes. Security design is operationally defined as the correlation between the two risky assets in the experimental markets. ‘Market outcomes’ include qualities such as price and allocational efficiency, but this research gives special attention to allocational efficiency. The original intention of this research was to conduct experiments consisting of trading sessions carried out by human subjects, however the ongoing pandemic has made it impossible to gather enough people in one physical location to run human trading sessions. As a result, this research instead focuses on results from simulations consisting of two mean-variance utility-optimizing ‘robots’ trading against each other. The primary hypothesis of this research is that markets consisting of securities that are negatively correlated will be more allocationally efficient. At first glance our results are somewhat mixed both in favor of and against the hypothesis, however some further analysis gives a clearer picture of what may be driving the results.

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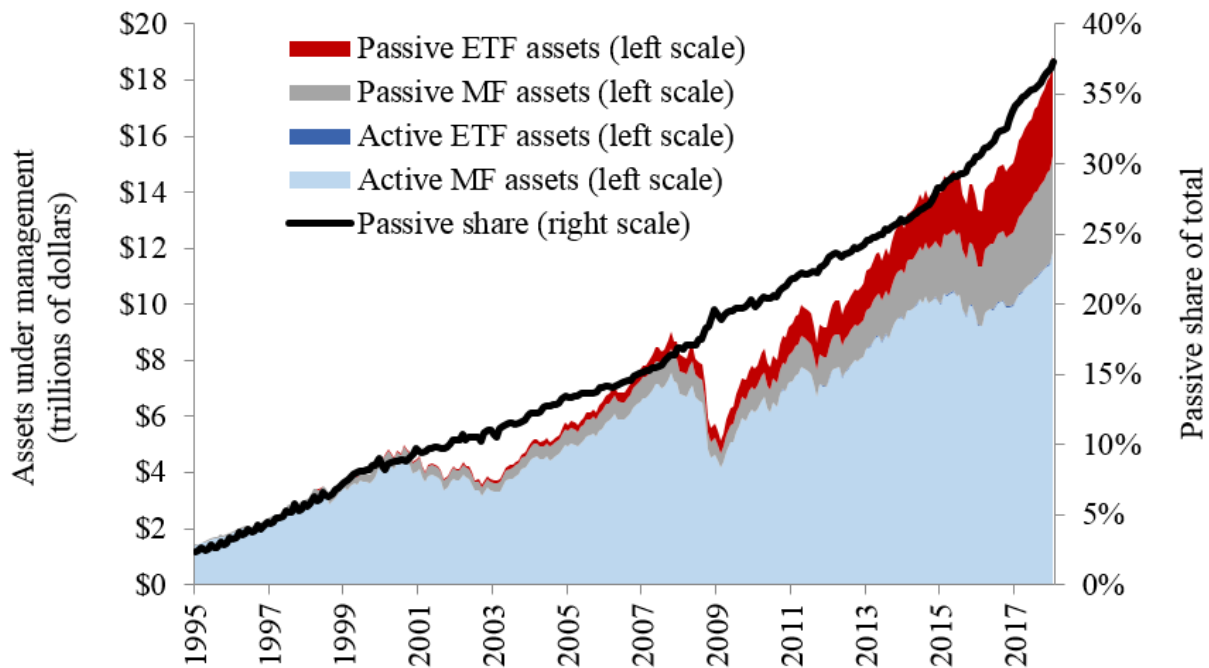
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INTRODUCTION

One of the most notable changes in prevailing investment principles to occur over the last several decades is the rising popularity of a passive investment style. Low-cost, highly diversified securities such as index mutual funds and exchange-traded funds (ETFs) have received high praise in recent years as one of the best methods available to the average investor looking to grow their wealth over time.

Eugene Fama's introduction of the efficient market hypothesis (EMH) in 1970 and the extensive empirical testing of the theory that followed (see Carhart (1997), Fama and French (2010), and Busse et al. (2014) for examples) and provided support of the theory has likely played a large role in spurring the shift. Grossman (1976) conducts experiments demonstrating the ability of markets to aggregate information by collecting all individuals' private signals and forming a resulting market price that is itself more informative of fundamental value than each individual's private signal. Increasing numbers of investors have decided that paying exorbitant fees to managers to actively invest their funds is usually not worth it when the option to buy an index and earn the market return at low or no cost is available. A report completed by the Federal Reserve gives evidence of the shift, writing that "As of March 2020, U.S. stocks held in passive [mutual funds] and ETFs accounted for about 14 percent of the domestic equity market, up from less than four percent in 2005" (Anadu et al. 2020). Another report conducted by BlackRock in 2017 concluded that, at the end of 2016, 18% of global equity was owned by passive investors (BlackRock 2017). Bogle (2016) also provides a useful overview of this shift. See Figure 1 for a visual representation of this shift.

Figure 1. Total assets in active and passive MFs and ETFs and passive share of total from 1995 to 2018. Reprinted from: McCabe, Patrick. "The Shift from Active to Passive Investing: Potential Risks to Financial Stability?" Harvard Law School Forum on Corporate Governance. 29 Nov., 2018. Source: Morningstar, Inc.



One observation to be made in regards to the shift is that index funds tend to be much more highly positively correlated with each other than other types of securities. It is essential to think about how the changing nature of assets held globally changes the types and magnitudes of benefits (or costs) that the financial system is capable of providing for/imposing on society. This research contributes to understanding of how markets consisting of more positively correlated assets may impact their ability to allocate resources efficiently. We use experiments to examine market equilibration processes and outcomes as a result of imposed fundamentals and specifically to focus on what types of security design lead to the best (most efficient) allocational outcomes. Security design is defined here as the correlation between assets in the market.

The rise in popularity of securities such as ETFs and index funds that are more conducive to a passive investment strategy has already gained extensive attention from academics, practitioners, and other interested parties. However, the vast majority of work done so far has been empirical in nature. Empirical methods are inherently unable to comprehensively (or convincingly) answer all questions related to understanding what this new composition of assets in global markets means for policy, regulation, etc. Specifically, while empirical methods are often used to address questions of price efficiency in markets, these methods are severely ill-suited to address another equally important component of market effectiveness – allocational

efficiency. Allocational efficiency refers to the idea that all market participants have optimized their holdings with respect to their beliefs/preferences. A market participant's beliefs and preferences include characteristics such as attitudes towards risk, attitudes towards ambiguity, beliefs about current or future states of the world (possibly based on signals a participant may have received that could have been public or private), etc. Allocational outcomes are hoped to be Pareto-optimal, implying that no agent can be made better off without simultaneously making another agent worse off. Allocational efficiency cannot be tested using field data as the researcher has no access to the composition of individual portfolios, and even less so to the preferences of individuals. The central question of this research is what impact does the correlation between assets in a market have on the efficiency of the given market, with special attention given to allocational efficiency.

To elaborate further on the appropriateness of an experimental approach to answer the questions posed here, consider that there is substantial evidence and agreement suggesting that there are forces at work in markets which drive them to equilibrium (see, for example, Arrow and Hahn (1971)). There is much less agreement, however, on what these forces are that drive markets to equilibrium. Furthermore, it is very difficult to learn about these driving forces through the analysis of historical data because while the price data is of high quality, not enough is known about the fundamentals of past markets. This represents a great opportunity for experimental finance, where markets can be created in a laboratory setting. In these laboratory markets, researchers can not only know what the fundamentals are of the markets they create but also control and change them to see if outcomes change. As a result, the laboratory can produce counterfactual evidence, an impossibility in empirical data. It will thus allow us to ask questions not only about price and allocational efficiency but also about what types of security structures best facilitate achieving those efficiencies. In particular, experiments make it possible to create market structures with varying degrees of correlations between securities, *while keeping all else the same*, and thus isolate the effect of correlations on efficiency.

One result of general equilibrium theory proposed by Walras in *Elements of Pure Economics* is that the correlation between assets should not affect the ability of markets to reach a Pareto-efficient equilibrium. The extant theory regarding efficiency of free markets posits that the only conditions needed to support an efficient equilibrium are locally non-satiated preferences, complete markets, and perfect information. This would suggest that the correlation

between assets in the market have no impact on the ability of the market to reach equilibrium. That being said, a recent paper by Asparouhova, Bossaerts, and Ledyard has produced results in support of a more nuanced *equilibration* theory in which asset structure has an effect on allocational outcomes. In particular, the equilibration dynamics imply that the path to equilibrium depends on the variance-covariance matrix between assets. The experiments discussed in this paper ask whether or not this theory of allocational efficiency as related to security design will be supported. Should the experiments end up supporting the hypothesis, the project could yield substantial policy implications regarding what types of security designs lead to optimal allocational outcomes.

This research uses a trading platform called Flex-E-Markets to create Continuous Double Auction (CDA) markets consisting of two risky securities (A and B) and a riskless security (cash). The CDA setting is used for its evidenced ability (as in Smith (1962)) to support convergence to equilibrium in single commodity markets as well as in multiple-asset markets (see also Asparouhova, Bossaerts, and Plott (2003), for example), which is largely a result of its open book and consequent shared information. The theoretical (equilibrium) framework of this research is that of the Capital Asset Pricing Model (CAPM) developed in Sharpe (1964), Lintner (1965), and Mossin (1966). This framework is ideal mostly for its ability to price risky assets according to the extent to which these assets co-vary with aggregate risk in the market (as evidenced in experiments in Bossaerts and Plott (2004)). In this way, the equilibrium framework is in line with the most important features of modern asset pricing theory.

In this research we report results from simulations in two-person economies with two risky assets and cash. Individuals are differently endowed with the two assets – called “A” and “B” – and cash, and are then invited into a marketplace to trade with each other. Imposition of the mean-variance utility that results from CAPM encourages the individuals to trade away the idiosyncratic risk inherent in their respective endowments. Keeping state-contingent endowments and individual preferences the same, the simulated agents participate in three different treatments defined by the security’s payoffs, and more importantly, their covariances. Equilibration paths and resulting market outcomes can then be compared across treatments to examine differences that arise as a result of varying the security design. The original intention of this research was to examine the impact of security design on market outcomes by having human subjects participate

in trading in these markets in a controlled laboratory setting. The envisioned economy was the replica of the two-person economy¹.

Simulations using mean-variance optimizing (MVO) trading agents that are described in more detail in later sections of this research were originally intended to be used as a proof of concept and to develop preliminary results before actual sessions with human subjects were conducted. However, the ongoing global pandemic has made it impossible to gather enough human subjects into one physical location as would be required to conduct the desired experiments. Due to this, the focus of this research was re-directed towards the simulations using the algorithmic trading agents. Results from these simulations are what will be discussed in the remaining sections of this paper.

The next section includes a review of related literature, which is followed by a description of the experimental design given to help illustrate the theoretical framework, which is then explained more generally in the section after, results are then given, and finally some concluding discussion finishes.

RELATED LITERATURE

This research uses an experimental approach to further understanding of how the correlation between assets in a market may affect the equilibrium achieved by such a market and the path, both of prices and holdings of the assets, that the market may follow to reach said equilibrium. In doing so, this research also adds to the large body of literature that tries to understand the potential impacts on market outcomes of the increasing popularity of securities that are commonly used as part of passive investing strategies such as index mutual funds and ETFs. The first index fund was created in 1975 by John Bogle, making this type of security now 45 years old in the year (2020) in which this is being written. Although such investing practices have not become largely popular until more recent years, academics and other interested parties have dedicated substantial efforts to researching the topic.

For example, Brogaard, Ringgenberg, and Sovich (2019) gives empirical evidence of the real economic impact that index investing may have. The research focuses on commodity

¹ Debreu and Scarf (1963) define the replica economy as one that has a finite number of agents of the types of the original economy. In the case of a replica of a two-person economy, this implies having equal number of agents of the two types.

indices, and uses a difference-in-differences framework applied to the spike in popularity in commodity index investing that occurred around 2004 to examine the impact that the spike had on different firms. They refer to this rise in popularity of commodity index investing as the “financialization” of commodity markets. The authors try to isolate the effect that index investing has on firms by dividing all firms included in the study into two groups: index commodity firms and non-index commodity firms. Index commodity firms are defined as firms with substantial exposure to index commodities, as a result of these firms using index commodities heavily in their business. The research finds that “following financialization, index commodity firms experience a 6% increase in costs and a 40% decrease in operating profits relative to non-index commodity firms.”

Brogaard et al. (2019) also tries to understand the mechanisms underlying the harm that index investing seems to do to firms. The research focuses on two main channels through which the negative impact may arise: a budget constraint channel and a feedback channel. The budget constraint channel can be summarized by the intuition that increased index investing in a commodity causes higher prices and volatility for the commodity, which in turn forces firms that use the commodity to either raise prices or accept lower profit margins, either of which will decrease profits for the firm. The feedback channel, as explained by the authors, “argues that financialization impacts firm performance because it changes the informational content of prices, which then impacts firm production decisions.” Their results provide evidence suggesting that both channels play an important role in the harm done to firms by index investing.

Sushko and Turner (2018) discusses the rising popularity of passive investing as evidenced by recent outflows of funds from actively managed mutual funds compared to stable passive mutual fund flows. The work also discusses possible implications of this shift related to pricing capabilities of markets that are increasingly inhabited by passive investors. Lastly, the work highlights the link between this research and the passive investing literature by noting the potential for this shift to result in markets consisting of assets that are more positively correlated.

Many of the attempts to study the potential impacts that rising popularity in passive investing may have on the ability of markets to effectively allocate resources focus on price efficiency, or the ability of a given market to price assets according to their underlying fundamental values. This is a logical place to start given the intuition that rising numbers of passive investors in markets may mean fewer active participants conducting the costly but

necessary information acquisition needed to keep prices accurate. A subset of the index investing literature focuses on commodity markets to try and understand potential impacts on prices and volatility, with varying results. Some of this work, such as Stoll and Whaley (2010) and Hamilton and Wu (2015) find that increased index investing in commodity markets did not impact the prices and volatility of the underlying commodities. Other work in the subfield, however, such as Singleton (2015) and Henderson et al. (2015), do find evidence of such price effects.

Examples of empirical work on impacts of index investing not limited to commodity markets also abound. Irwin and Sanders (2011) speaks to the possible impact on price efficiency that index funds may have in their examination of the role that such funds played in the 2007-2008 asset pricing bubble. Ben-David et al. (2013) provides empirical evidence suggesting that ETFs increase the intraday and daily volatility of the individual stocks of which they are made up. Wermers and Yao (2010) finds that “active funds increase the price efficiency of stocks through their trades” and also “that stocks with ‘excessive’ levels of passive fund ownership and trading exhibit more long-term pricing anomalies as well as a larger price reversal following trades.” Israeli et al. (2017) comes to similar conclusions regarding the potential implications that increased passive investing may have for price efficiency. Lastly, Coles, Heath, and Ringgenberg (2018) finds that “while index investing changes investor composition and information production, it does not alter price informativeness.” Note, importantly, that all studies mentioned so far are of an empirical nature and also that they mostly focus on the price efficiency of markets. Unfortunately, empirical studies are unable to examine all the ways in which passive investing and higher positive correlation among assets may impact the effectiveness of markets.

The difficulties associated with testing allocational efficiency empirically represent a great opportunity for experiments. Early work on experimental asset markets such as Plott and Sunder (1982), Forsythe et al (1982), and Friedman et al. (1984) focus on fundamental topics such as whether or not prices converge to their expected equilibrium in experimental markets and the ability of market participants to effectively smooth their consumption via participation in trade. Carbone et al. (2020) is an example of recent research that builds on these early works, focusing on experimental asset market outcomes such as price efficiency in different types of asset markets. Specifically, the work compares market outcomes between a long-term asset

market and a short-term credit market. The two markets are theoretically equivalent, however the short-term credit market delivers better pricing results, and, importantly, significantly better allocational efficiency results. The authors hypothesize that pricing the short-term credit instrument as well as using it for consumption planning is cognitively less taxing than using a long lived consol bond. Crockett et al. (2019) also conducts asset pricing experiments, focusing on parsing out what types of assets incentivize participants to more effectively smooth their consumption over time. Asparouhova, Bossaerts, and Ledyard (2019) (ABL from now on) and Bossaerts, Plott, and Zame (2007) show that, in comparison to price efficiency, there is less certainty that markets achieve allocational efficiency in a timely manner. Such results highlight the need for further research on the topic. An important step towards understanding how markets might improve the speed and accuracy with which they move towards allocational efficiency is understanding the forces at work in a market that drive the equilibration process. ABL also produced results suggesting that allocational outcomes are closer to Pareto-optimal when securities are structured to correlate negatively. This finding is largely what drove the main hypothesis of this project.

EXPERIMENTAL DESIGN

This study focuses on 3 treatments: positive, negative, and zero correlation between the two risky assets in the market. Treatments are indexed by j for $j = \{negative (neg), zero, positive (pos)\}$. Note that assets are defined by their payoffs in each of three potentially realizable states: X , Y , and Z , each of which occur with equal probability ($1/3$). An asset that does not have the same payoff in all three possible states is deemed risky. Tables 1-3 are given below and describe the payoffs of the assets in each of the three treatments, with the correlation between A and B in treatment j , denoted $\rho_{j,AB}$, given directly below each table. Notice that the payoffs of Risky Asset A do not change across the three treatments. Rather, the payoffs of A are held constant across treatments while the payoffs of B are manipulated to induce the desired correlation between A and B for each treatment. Also note that cash always pays out 1 regardless of state, treatment, etc. (cash is riskless). In all treatments, all assets have an expected payoff of \$1.

Table 1. Asset Payoffs (negative structure)

State	X	Y	Z
Stock A	\$2	\$0	\$1
Stock B	\$0.50	\$1	\$1.50
Cash	\$1	\$1	\$1

$$\rho_{neg,AB} = -0.5$$

Table 2. Asset Payoffs (zero structure)

State	X	Y	Z
Stock A	\$2	\$0	\$1
Stock B	\$1.50	\$1.50	\$0
Cash	\$1	\$1	\$1

$$\rho_{zero,AB} = 0$$

Table 3. Asset Payoffs (positive structure)

State	X	Y	Z
Stock A	\$2	\$0	\$1
Stock B	\$1	\$0.50	\$1.50
Cash	\$1	\$1	\$1

$$\rho_{pos,AB} = 0.5$$

Once asset structures producing the desired correlations for each treatment were arrived at, endowments of these assets for each individual type were left to be determined. These asset endowments also vary for each individual across the treatments. This is done deliberately to keep the Arrow-Debreu endowments (endowed wealth) constant across all three treatments. Arrow-Debreu (AD) endowments are the distributions of wealth across the three states of nature. Further explanation of how this wealth-endowment equivalence is achieved and its importance is given in the following section. Tables 4-6 given below describe asset endowments for each individual Type in each treatment.

Table 4. Initial asset endowments (negative structure)

Type	Asset A	Asset B	Cash
I	100	200	\$160
II	220	120	\$120
Market	160	160	\$140

Table 5. Initial asset endowments (zero structure)

Type	Asset A	Asset B	Cash
I	50	-100	\$510

II	190	-60	\$330
Market	120	-80	\$420

Table 6. Initial asset endowments (positive structure)

Type	Asset A	Asset B	Cash
I	0	200	\$260
II	160	120	\$180
Market	80	160	\$220

A more general explanation of Arrow-Debreu (AD) equivalence is given in the following section but consider the following specific example to help illustrate for the time being. Focus on individual 2's endowment in the positive treatment. Their endowment of 160 units of A provides a state contingent wealth of 320, 0, 160 in states X, Y, and Z, respectively. Their endowment of 120 units of asset B provides wealth of 120, 60, 180 in each of the three states. These endowments, combined with their endowed cash of \$180 (with invariant pay across states), provides individual 1 endowed wealth of 620, 240, and 520 in states X, Y, and Z (respectively). The vector (620, 240, 520) is individual 1's AD endowment. The endowments for each individual in each of the three treatments provided above all result in identical AD endowments. AD endowments are somewhat arbitrary but ultimately arrived at by some amount of trial and error in the process of matrix algebra described in the following section. AD endowments are somewhat arbitrary but ultimately arrived at by some amount of trial and error in the process of matrix algebra described in the following section. The important quality of the AD endowments that were ultimately settled on is that they create sufficient difference across individuals, giving enough room for utility gains from trading away from the AD endowments. The mean-variance preferences of the market participants incentivize them to complete trades that diversify away the risk inherent in the assets they are endowed. Enough trades take place between the individuals in each treatment to result in a non-trivial path to equilibrium from beginning to ending holdings. The paths followed in each treatment can then be compared against each other for the purposes of answering the central questions of this research.

Consistency with the theoretical framework used in our experiments requires that participating individuals seek to maximize the payoff mean and minimize the payoff variance of

their holdings. Normally, we induce mean-variance preferences in subjects by applying a mean-variance transformation of their final holdings to determine compensation for participating in experiments. However, as has been stated earlier, the results discussed in this research are limited to simulations that consisted of two ‘robots’ trading against each other. Because participants were not, in this case, actual humans, compensation was not necessary and thus mean-variance preferences were not induced in the typical manner. Instead, the mean-variance preferences of these trading robots are embedded in the algorithms according to which they are programmed to make trades. More general exposition of mean-variance preferences is again left for the next section, but consider the following example to help illustrate what is meant by the phrase. Take individual 2 from the prior example. Mean-variance preferences imply that this individual’s utility resulting from their endowment of 160, 120, and 180 in A, B, and cash respectively is equal to the mean across the three states of their consequent AD endowment of (620, 240, 520) minus a penalty for the variance. The penalty measures the degree of risk aversion of individual 2.

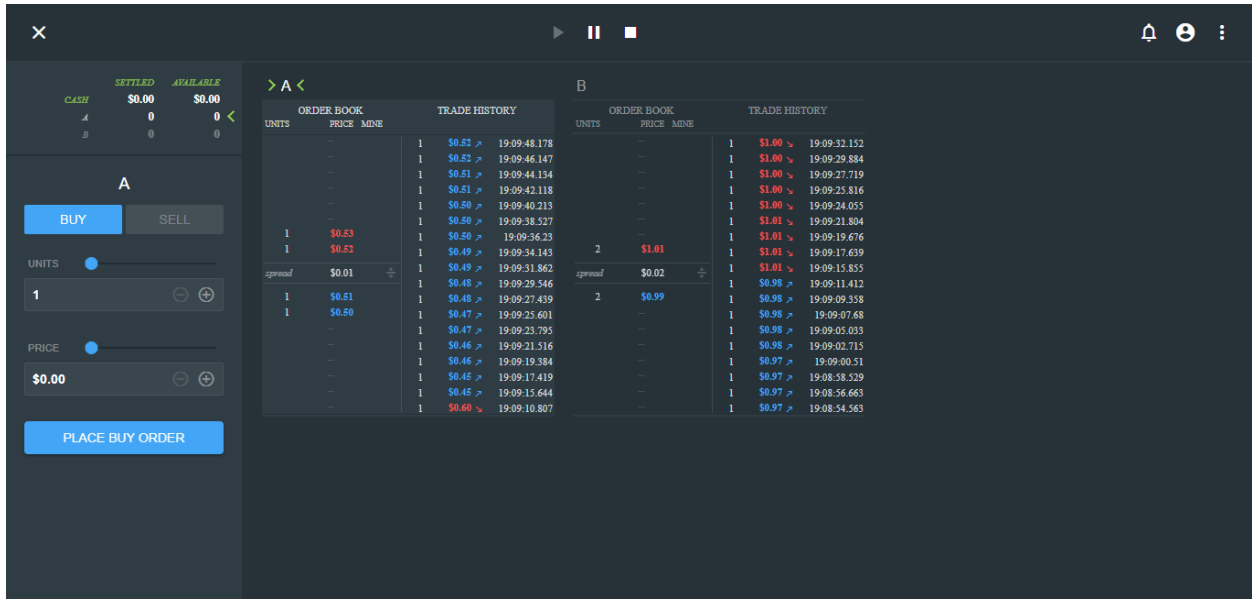
All robots used in simulations were Mean-Variance Optimizing (MVO). These agents can be thought of as similar to the zero-intelligence agent developed in Gode and Sunder (1993, 1994) but extended to multiple markets. They follow a local optimization rule using current holdings, asset payoffs, a risk-aversion parameter, and a spread parameter as inputs, continuously computing marginal valuations for assets A and B based on mean variance utility functions. The robots are described as ‘zero-intelligence’ because they ignore price histories and instead use only their own marginal valuation (based on current holdings, asset payoffs, and a risk-aversion parameter) combined with their given spread parameter to determine prices at which they should post orders in the market. As an example, one of these zero-intelligence agents may calculate a marginal valuation for a given asset of \$1.00, and, given a spread of \$0.02, post a sell order for the asset at \$1.01 and a buy at \$0.99 (spread straddles the valuation), even though the last three times market participants have completed a transaction for the asset, the price has always been above \$1.50. The phrase ‘zero-intelligence’ is used to describe the idea that these MVO agents ignore the possibility that their own personal valuation for an asset at any given point in trading may be very low (or high) relative to the valuations of other market participants as evidenced in recent trades.

The risk-aversion parameter was not of particular interest for this research and was kept at 0.006 for both robots across all simulations and treatments. The value of 0.006 is only non-arbitrary to the extent that it produces reasonable (i.e. non-negative, non-huge) valuations for the range of holdings the robots could feasibly obtain based on the endowments of assets assigned to them across all treatments. The spread parameter is also not of particular interest for this research but is necessary in order for the robots to be able to function. Spread parameters were held constant at \$0.02 for both robots across all simulations and treatments.

MVO agents can be either market-making or market-taking. All MVO agents used in simulations for this research were maker MVOs. This means that at any given moment during a simulation, the robots are calculating their marginal valuations for assets A and B and then posting buy and sell orders for the assets (for one unit at a time) at prices determined by their spread and marginal valuations, hoping that the orders will be completed by another party. As soon as one of the orders that a robot has posted is completed, it will recalculate valuations and immediately post a new order to replace the one that has been consumed. In the context of the simulations run for this research, the robots were always trying to keep four orders on the market - one buy and one sell for each of the two assets. MVO agents used in simulations conducted for this research are restricted to posting single-unit buy and sell orders in accordance with their local-optimization nature. The process of applying a mean-variance utility function to these locally-optimizing agents in order to arrive at specific prices at which the agents post buy and sell orders is given in the following section.

Ten simulations were run for each treatment for a total of 30 simulations. Simulations were conducted by inviting two MVO agents into a marketplace to trade with each other. . All simulations were run using a software called Flex-E-Markets that allows users to set up markets and invite others into the market to participate in trading rounds. All markets were organized as continuous double auctions. The original intent for this research was to report results from trading rounds with human subjects, who would make trades manually via the Flex-E-Markets user interface (UI). See Figure 2 for an example screenshot of the Flex-E-Markets UI.

Figure 2. Flex-E-Markets UI (screenshot taken at the end of a simulation round after the two MVO agents had reached equilibrium)



Unfortunately, experimental sessions with human subjects were not possible for reasons already explained. Consequently, all results discussed in this research come from simulations that were run consisting of two MVO robots that traded via the Flex-E-Markets application program interface (API).

An important feature of the experimental design of this research was addressing the effect that which individual (the Type 1 individual or Type 2 individual) entered the market first in each simulation round had on the equilibration path in that round. See the concluding discussion of this research for elaboration on the importance of which individual enters the market first. To address this effect, simulation rounds alternated between the Type 1 individual starting first and the Type 2 individual starting first – odd-numbered rounds were ‘Type 1 first’ rounds and even-numbered rounds were ‘Type 2 first’. In any given round, the robot that starts first posts all four limit orders (one buy and one buy sell for each of the two assets) before the other robot begins posting orders.

THEORETICAL FRAMEWORK

In this research, treatments consist of varying the payoffs of asset B to induce varying correlations between asset A and asset B. Let d_j be the 2×3 matrix describing the payoffs (or dividends) of the two risky assets in each of the three states (x, y, and z) for treatment j . Asset (A or B) is denoted in the subscript and state in the superscript.

d_j is defined generally as:

$$d_j = \begin{bmatrix} d_{j,A}^x & d_{j,A}^y & d_{j,A}^z \\ d_{j,B}^x & d_{j,B}^y & d_{j,B}^z \end{bmatrix}$$

d_j has an associated variance-covariance matrix, denoted D_j , which can be written as:

$$D_j = \begin{bmatrix} D_{j,AA} & D_{j,AB} \\ D_{j,BA} & D_{j,BB} \end{bmatrix}$$

An element in D_j , say $D_{j,AB}$ for example, denotes the payoff covariance between assets A and B in treatment j (note that the payoff covariance of an asset with itself, such as $D_{j,AA}$, is just the payoff variance of the asset). Furthermore, the correlation between A and B, $\rho_{j,AB}$, can be written in terms of elements of the variance-covariance matrix as follows:

$$\rho_{j,AB} = \frac{D_{j,AB}}{\sqrt{D_{j,AA}D_{j,BB}}}$$

Of course, when the payoffs of the assets change in d_j , the resulting values in D_j change and ultimately $\rho_{j,AB}$ changes. This is the mechanism used to induce the desired correlations between the two assets in each treatment.

Recall from the prior section that individuals' endowments of assets change across treatments in a particular way that corresponds to the changing payoffs of the assets across treatments. To ensure that differences in allocational efficiencies between treatments are actually due to the correlation between the assets in each rather than due simply to the nature of endowments, each individual must have the same endowed wealth vector across each of the three treatments. It is useful at this point to introduce the notion of Arrow-Debreu securities. In the three-state economy used for the framework of this research, the set of AD securities is the simple spanning set of three securities, each of which pays out one in one state and zero in the other two, where the state in which each pays out one is unique to that security. Let the following three 1×3 matrices, denoted AD_1 , AD_2 , and AD_3 , represent the payoffs of AD securities one, two, and three in states x, y and z respectively.

$$AD_1 = [1 \quad 0 \quad 0]$$

$$AD_2 = [0 \quad 1 \quad 0]$$

$$AD_3 = [0 \quad 0 \quad 1]$$

AD_1 , AD_2 , and AD_3 may be bound by row, forming AD , which describes the payoffs of each of the three AD securities in each of the three states.

$$AD = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Let W_i represent the 1×3 matrix describing the wealth that an individual i is endowed with in each of the three states. Also let ADE_i represent the 1×3 vector describing the individual's endowed number of units of each of the three AD securities. Of course, both $ADE_i \times AD = W_i$ and thus $ADE_i = W_i$ must always be true.

We would like to keep the endowed *wealth* constant across treatments and the introduction of AD endowments helps to determine endowments of marketable securities A, B, and cash in each treatment that achieve this.

Let ADE_i denote the 1×3 matrix describing the desired endowed wealth (or AD endowment) for an individual i in $i=1,2$ in each of the three states. Also let \tilde{d}_j denote the 3×3 matrix (formed by binding d_j with $(1, 1, 1)$ by row) describing the payoffs of each asset (including cash) in each treatment j (described in table form in the experimental design (Tables 1-3)). Lastly, let AE_{ij} denote the 1×3 matrix describing the endowments of marketable assets (A, B, and cash) for individual i in treatment j . The AE_{ij} required to achieve the desired Arrow-Debreu endowments can be solved for as follows:

$$AE_{ij} \times \tilde{d}_j = ADE_i$$

$$AE_{ij} = ADE_i \times \tilde{d}_j^{-1}$$

The asset endowments described in Tables 4-6 provided in the experimental design section were arrived at using the method explained here combined with the desired Arrow-Debreu endowments listed in the following Table 7 and the previously determined asset payoffs in each of the three treatments. The last row of the table describes the market portfolio of Arrow-Debreu securities, which is simply the per capita endowed wealth.

Table 7. Arrow-Debreu endowments (constant across all 3 treatments)

Type	State X	State Y	State Z
I	460	360	560
II	620	240	520
Market	540	300	540

This research follows the lead of the majority of the vast experimental asset pricing literature in utilizing the Capital Asset Pricing Model (CAPM) as the guiding model to predict

and shape participant payoff incentives. In accordance with CAPM, agents that traded during the simulation rounds exhibited mean-variance preferences. The theoretical framework used in this research follows that of Asparouhova et al. (2019) and Asparouhova et al. (2020).

Let $e_{i,j}^0$ be individual i 's cash endowment and $e_{i,j} = (e_{i,j}^A, e_{i,j}^B)$ represent individual i 's endowment of risky assets A and B (respectively) in treatment j . Note that $AE_{i,j}$ defined above is equivalent to $(e_{i,j}^A, e_{i,j}^B, e_{i,j}^0)$. Also let $m_{i,j}$ denote individual i 's ending cash holdings and $x_{i,j} = (x_{i,j}^A, x_{i,j}^B)$ represent individual i 's final holdings of the risky asset assets A and B (respectively) in treatment j . Recall that d_j is the 2×3 matrix describing the payoffs (or dividends) of the two risky assets in each of the three states in treatment j and D_j its associated 2×2 variance-covariance matrix. Also let \bar{d} be the vector of expected payoffs for assets A and B (the subscript indexing treatment is left off of \bar{d} because expected payoffs are always 1 for assets A and B across all treatments).

Subject i 's final wealth in treatment j can be written $m_{i,j} + d_j^T x_{i,j}$ which represents the sum of their ending cash holding plus dividends received from their ending holdings of risky assets A and B. Application of a mean-variance utility function to final wealth can be written as

$$U_{i,j}(m_{i,j}, x_{i,j}) = m_{i,j} + \bar{d}^T x_{i,j} - \frac{\gamma_i}{2} x_{i,j}^T D_j x_{i,j}. \quad (1)$$

Note that γ_i denotes individual i 's risk aversion (which is constant across all treatments).

Market Equilibrium

Given asset prices $p = (p^A, p^B)$ for risky assets A and B, respectively, maximizing (1) subject to the budget constraint $m_{i,j} \leq e_{i,j}^0 + p^T (e_{i,j} - x_{i,j})$ yields a demand function for individual i in treatment j that can be written as

$$x_{i,j} = \frac{1}{\gamma_i} D_j^{-1} (\bar{d} - p). \quad (2)$$

Assuming that the market clears implies that the sum of final holdings among all market participants must be equal to the sum of all initial endowments among all individuals and subsequently leads to the following equilibrium prices in treatment j :

$$p_j^* = \bar{d} - \frac{1}{\sum_{i=1}^I \frac{1}{\gamma_i}} D_j \sum_{i=1}^I e_{i,j} = \bar{d} - \Gamma D_j \bar{e}_j. \quad (3)$$

Where $\Gamma = \frac{1}{\sum_{i=1}^I \frac{1}{\gamma_i}}$ represents the harmonic mean risk aversion and $\bar{e}_j = \sum_{i=1}^I e_{i,j}$ represents the per capita endowment of assets which we also call the market portfolio in treatment j .

We would also like to show that equilibrium wealth is equal across all treatments. We assume that individuals reach the market portfolio \bar{e}_j . To establish equilibrium holdings of cash, we also assume that every transaction that individuals participate in to change their holdings of risky assets from their initial endowments of $e_{i,j}$ to the market portfolio \bar{e}_j occurs at equilibrium prices p_j^* . With this, an individual's change in cash $\Delta e_{i,j}^0$ can be written as:

$$\Delta e_{i,j}^0 = (p_j^*)^T \times (e_{i,j} - \bar{e}_j)$$

Their ending cash $e_{i,j}^f$ is their endowment of cash $e_{i,j}^0$ plus their change in cash $\Delta e_{i,j}^0$:

$$e_{i,j}^f = e_{i,j}^0 + \Delta e_{i,j}^0$$

Ending cash $e_{i,j}^f$ may then be concatenated with the transpose of the market portfolio \bar{e}_j to form vectors describing each individuals' holdings of A, B, and cash at the end of a round, denoted $h_{i,j}$. Finally, the vector describing an individual's equilibrium wealth in each of the three states, denoted w_i , at the end of a round may be written as the simple matrix multiplication of their ending holdings with the treatment's associated matrix of asset (A, B, and cash) payoffs:

$$w_i = h_{i,j} \times \bar{d}_j$$

w^* is used to denote the set of w_i for $i = \{1,2\}$ that occurs in equilibrium. Note that w^* characterizes an Arrow-Debreu equilibrium, and also that w^* does not change across treatments (because Arrow-Debreu *endowments* do not change across treatments). See Proposition 19.D.1 in Mas-Colell (1995) for explanation of Radner equilibriums and their corresponding Arrow-Debreu equilibriums (and vice versa). Across all treatments, \bar{d} , $\sum_{i=1}^I \frac{1}{\gamma_i}$, w_1 , and w_2 are given by:

$$\bar{d} = \begin{bmatrix} 1 \\ 1 \end{bmatrix}, \sum_{i=1}^I \frac{1}{\gamma_i} = \frac{I}{0.006}, w_1 = [568.8 \quad 328.8 \quad 568.8], w_2 = [511.2 \quad 271.2 \quad 511.2].$$

In the negative correlation treatment, D_{neg} and $\sum_{i=1}^I e_{i,neg}$ are as follows:

$$D_{neg} = \begin{bmatrix} \frac{2}{3} & -\frac{1}{6} \\ -\frac{1}{6} & \frac{1}{6} \end{bmatrix}, \sum_{i=1}^I e_{i,neg} = I \times \begin{bmatrix} 160 \\ 160 \end{bmatrix}.$$

This results in predicted equilibrium prices in the negative treatment of \$0.52 for asset A and \$1.00 for asset B. Consequently, $h_{1,neg}$ and $h_{2,neg}$ are as follows:

$$h_{1,neg} = [160 \quad 160 \quad 168.8], h_{2,neg} = [160 \quad 160 \quad 111.2].$$

In the zero correlation treatment, D_{zero} and $\sum_{i=1}^I e_{i,zero}$ are as follows:

$$D_{zero} = \begin{bmatrix} \frac{2}{3} & 0 \\ 0 & \frac{1}{2} \end{bmatrix}, \sum_{i=1}^I e_{i,zero} = I \times \begin{bmatrix} 120 \\ -80 \end{bmatrix},$$

and resulting equilibrium prices are \$0.52 for asset A and \$1.24 for asset B. Resulting $h_{1,zero}$ and $h_{2,zero}$ are:

$$h_{1,zero} = [120 \quad -80 \quad 448.8], h_{2,zero} = [120 \quad -80 \quad 391.2].$$

In the positive correlation treatment, D_{pos} and $\sum_{i=1}^I e_{i,pos}$ are as follows:

$$D_{pos} = \begin{bmatrix} \frac{2}{3} & \frac{1}{6} \\ \frac{1}{6} & \frac{1}{6} \end{bmatrix}, \sum_{i=1}^I e_{i,pos} = I \times \begin{bmatrix} 80 \\ 160 \end{bmatrix},$$

and resulting equilibrium prices are \$0.52 for asset A and \$0.76 for asset B. Consequently, $h_{1,pos}$ and $h_{2,pos}$ are as follows:

$$h_{1,pos} = [80 \quad 160 \quad 248.8], h_{2,pos} = [80 \quad 160 \quad 191.2].$$

Note that, across all three of the treatments, the following equalities hold:

$$w_1 = h_{1,j} \times \tilde{d}_j, w_2 = h_{2,j} \times \tilde{d}_j.$$

Mean-Variance Utility Optimizing Robots

For each of the two assets in $s = \{A, B\}$, an MVO agent will calculate the reservation price for asset s as $\rho_{si}^t = \frac{\partial u_i(m_i^t, x_i^t) / \partial x_{is}^t}{\partial u_i(m_i^t, x_i^t) / \partial m_i^t}$. Note that this is also the marginal rate of substitution between asset s and cash for individual i^2 .

The MVO agents use their marginal valuation combined with their inputted spread parameter to determine values at which to post orders and can post buy and sell orders based on these values as follows:

$$b_{is,buy}^t = \rho_{is}^t - \delta_{is}^t \text{ and } b_{is,sell}^t = \rho_{is}^t + \delta_{is}^t.$$

After applying mean-variance utility to calculate ρ_{is}^t , this becomes:

$$b_{is,buy}^t = \bar{d}_s - \gamma_i(D_{ss}x_{is} + D_{ss'}x_{is'}) - \delta_{is}^t \text{ and } b_{is,sell}^t = \bar{d}_s - \gamma_i(D_{ss}x_{is} + D_{ss'}x_{is'}) + \delta_{is}^t$$

² For this research $\frac{\partial u_i(m_i^t, x_i^t)}{\partial m_i^t} = 1$ always holds making ρ_{si}^t the marginal value of asset s at time t

The central goal of this research is to vary the correlation between assets in a market and examine how equilibration paths and outcomes change as a result. The main hypothesis is that markets consisting of securities that correlate negatively will exhibit the highest allocational efficiency. The first measure used to compare the efficiency of markets across treatments is the total number of trades that markets in each treatment took to reach equilibrium. The implication of this basic metric is that a market that requires fewer trades to reach equilibrium, i.e. reaches equilibrium more quickly, is likely a more efficient market than one that requires more trades to reach equilibrium.

RESULTS

See Table 8 for a summary of the number of trades each simulation round took to reach equilibrium and means for each treatment.

Table 8. Number of trades to equilibrium

	treatment		
	negative	zero	positive
1	117	88	147
2	102	88	147
3	111	88	145
4	102	88	145
5	111	89	147
6	102	88	147
7	111	89	147
8	102	89	147
9	117	89	145
10	107	89	147
means	108.2	88.5	146.4

This basic metric does not fully support the initial hypothesis of this research (that markets consisting of negatively correlated assets are more efficient). While these results partially support the hypothesis because the positive treatments, on average, take the largest number of trades to reach equilibrium, they are also somewhat contradictory in that the zero treatment takes fewer trades on average than the negative treatment.

However, this first metric ignores an important difference across the treatments. Due to the varying asset endowments required across treatments to keep Arrow-Debreu endowments constant, looking at only the total number of trades in each treatment may not accurately portray the efficiency of the markets in question. This is because when the endowments of assets vary, the beginning and ending (equilibrium) marginal valuations (MVs) of the assets also change. For example, in the negative treatment, MVs for the Type 1 individual begin at \$0.80 and \$0.90 for assets A and B respectively and \$0.24 and \$1.10 for the Type 2 individual. In the equilibrium of this treatment, both individuals have MVs of \$0.52 and \$1.00 for A and B. Consequently, the MVs in this treatment in asset A must go from \$0.80 to \$0.52 for Type 1 and \$0.24 to \$0.52 for Type 2, meaning each individuals' MVs must change by \$0.28. In asset B, each individuals' MVs must change by \$0.10. This change in MVs required in each treatment can be thought of as the 'MV distance travelled.' In the zero correlation treatment, the MV distance travelled in asset A is the same, but for asset B it is substantially smaller at only \$0.06. A more revealing way to look at the trade numbers in each treatment is to first break the trade numbers down by asset and then also to scale these numbers by the MV distance travelled in each asset for each treatment. See Table 9 for a full report of these numbers.

Table 9. Trade numbers to equilibrium (by asset)

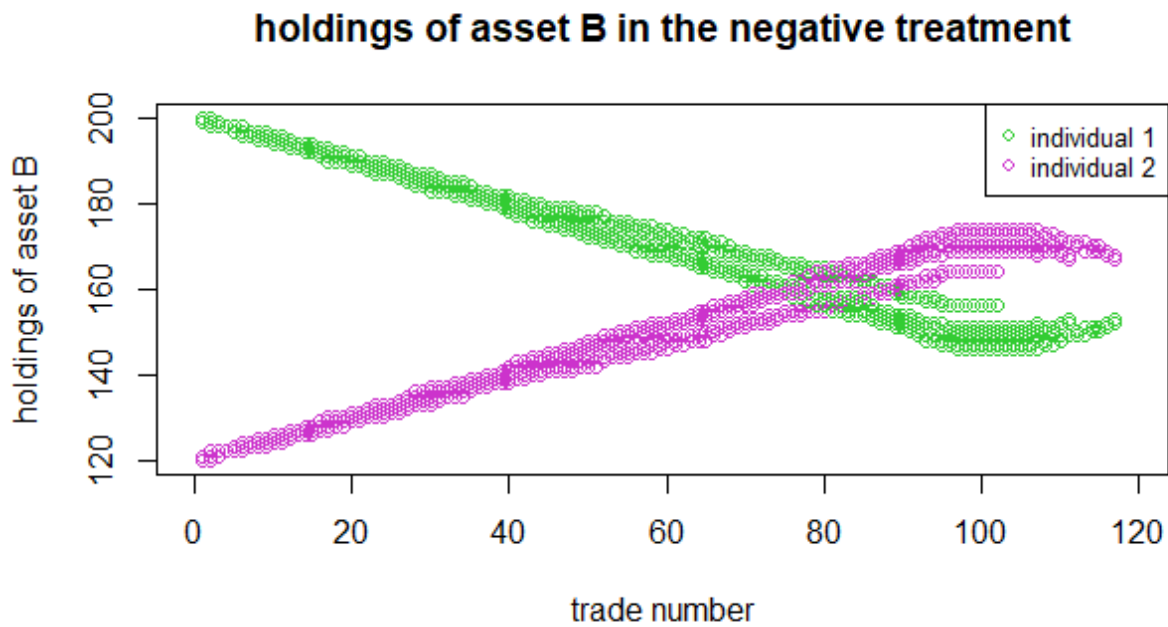
		treatment					
		negative		zero		positive	
		asset A	asset B	asset A	asset B	asset A	asset B
round	1	57	60	69	19	78	69
	2	58	44	69	19	78	69
	3	57	54	69	19	77	68
	4	58	44	69	19	77	68
	5	58	53	69	20	78	69
	6	58	44	69	19	78	69
	7	57	54	69	20	78	69
	8	58	44	69	20	78	69
	9	58	59	69	20	77	68
	10	57	50	69	20	78	69
	means	57.6	50.6	69	19.5	77.7	68.7
	MV distance (in cents)	28	10	28	6	28	4
	mean # trades per cent in MV distance	2.057	5.060	2.464	3.250	2.775	17.175

Accounting for MV distances in this way provides a clearer look at the efficiency of the path followed for each asset in each treatment. For example, the MV distance travelled in asset A is the same across all treatments at \$0.28, but the average number of trades to cover this distance in the negative treatment is only 57.6, while in the zero treatment it is 69 and in the positive 77.7. The last row of Table 9 reports the average number of trades to travel one cent in MV distance for each asset in each treatment. A lower number of trades required to travel one cent in MV distance implies a more efficient equilibration path. This metric almost fully supports the hypothesis, the only disparity being that average number of trades for asset B in the zero treatment (3.250) is smaller than that of the negative treatment (5.060).

One last way to help compare the relative efficiencies of the equilibration paths followed across treatments is to look at how holdings of the risky assets change through the equilibration process for both individuals across treatments. At first, one might think that changes in holdings over the course of a given round would be straight forward to predict. For example, in the negative treatment, for risky asset A, individual 1 is endowed with 100 units and individual 2 is endowed with 220 units. In equilibrium, they are predicted to split the market supply evenly, both holding 160 units of the asset. It would seem logical, then, that individual 1 would buy approximately 60 units from individual 2 (approximate due to the fact that most rounds fell some amount short of predicted equilibrium, for reasons to be explained in the next section), and then the individuals would be done trading in asset A for the round. This is roughly what happened for asset A in all three treatments, and it also happened for asset B in the zero treatment. It is not, however, what happened for asset B in the negative and positive treatments. In the negative treatments, individual 1 is endowed with 200 units of asset B, individual 2 is endowed with 120 units, and they are both predicted to hold 160 units in equilibrium. One would expect the individuals' holdings of asset B to move towards equilibrium in a linear manner, with each round consisting only of individual 1 selling units of B to individual 2 until they reach equilibrium. Instead, each round starts as expected, with individual 1 selling to 2, but in every round this continues *past* the predicted equilibrium point in which each individual holds 160 units. In every round, the individuals eventually worked to undo the overshooting of equilibrium that had occurred by switching the direction of transactions, with individual 1 buying units back from individual 2. In round 1, the round in which this overshooting of equilibrium happened most

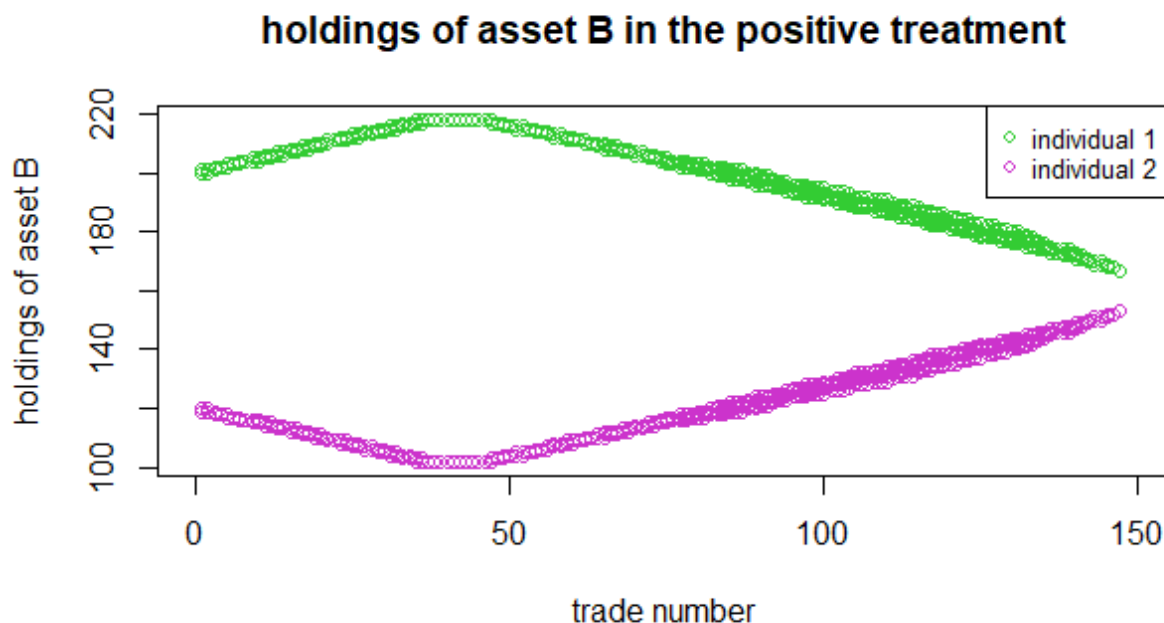
severely, individual 1 got down to only 146 units of asset B before starting to buy units back, reaching an ending holding of 152 units. See Figure 3 for a graph of each individual's holdings of asset B in all ten of the negative treatment rounds. Note that the trade numbers against which holdings are plotted include transactions in both asset A and asset B. This is why the slope at some points in each individual's plotted holdings is zero; at these points the trades occurring between the two individuals are in the other asset (asset A in this case), meaning that for these trades holdings of B are not changing.

Figure 3. Holdings of asset B in the negative treatment (with 'trade number' on x-axis including trades in A and B).



A similar pattern occurs in the positive treatment for asset B, but the effect is even more pronounced. In this treatment, individuals 1 and 2 are endowed with 200 and 120 units of asset B, respectively. In equilibrium, they both hold 160 units. The most direct path to equilibrium in asset B would consist of individual 1 selling 40 units of B to individual 2. This is not what happens in the ten simulation rounds. Instead, every round starts with individual 1 *buying* more B from individual 2, until individual 1 holds 218 units of B. At this point, the movement away from equilibrium stops, and individual 1 starts selling units of B back to individual 2, until they reach equilibrium, with individual 1 holding either 167 or 168 units of B.

Figure 4. Holdings of asset B in the positive treatment (with ‘trade number’ on x-axis including trades in A and B).



The fact that the two market participants would begin trading away from equilibrium right from the beginning of a round seems significant. Individual 1 starts with substantially more B than individual 2 but still has a higher marginal valuation for the asset (as evidenced by their willingness to buy units from individual 2). This must be a result of the individuals' relative holdings and valuations of the other assets. Any trading that occurs in a round that is in a direction away from equilibrium is worthy of mention in the context of this research for its seemingly obvious inefficiency. That being said, it is not fully understood why this trading away from equilibrium occurs in some treatments and not in others and whether or not the reason has to do with the varying correlation between assets A and B among the treatments. See Appendix B for graphs of holdings of A in all three treatments and B in the zero treatment. In these assets/treatments, trades that move holdings away from equilibrium do not occur, meaning that holdings follow the (expected) approximately linear path from endowments to equilibriums.

Another line along which allocational efficiency can be measured is proximity to theoretical equilibrium. In each of the three treatments, the theoretical framework described in an earlier section gives predicted equilibrium holdings that both individuals are trying to reach. Due

to the nature of the experimental design, however, individuals are usually not able to reach exactly these predicted equilibrium holdings. More specifically, this is a result of the fact that the smallest denomination in which orders can be posted in the Flex-E-Markets software is one cent. This means that the Type 1 and Type 2 individuals are forced to stop trading once their MVs are equal (plus a spread) for both assets. The fact that traders can only trade until their MVs are within one cent (half their spread of \$0.02) of each other's rather than until they are exactly equal means that they usually fall some number of transactions short of the predicted equilibrium. For example, predicted ending holdings in the negative treatment are $\bar{e}_{neg} = \begin{bmatrix} 160 \\ 160 \end{bmatrix}$ for assets A and B respectively for both individuals. However, many of the negative treatment rounds end with holdings such as $\begin{bmatrix} 157 \\ 152 \end{bmatrix}$ for individual 1 and $\begin{bmatrix} 163 \\ 168 \end{bmatrix}$ for individual 2. What is interesting and potentially meaningful is that there seems to be substantial difference in how close, on average, individuals get to predicted equilibrium holdings across each treatment. Markets that are able to get closer to their predicted equilibrium conditions have necessarily reached a more efficient outcome than markets that do not get as close. Think of efficiency here as the maximization of collective utility between the two individuals in the market. The predicted equilibrium is always the *most* efficient outcome possible, and the trading agents are programmed such that efficiency is strictly monotonically increasing with respect to trades as a result of the way efficiency is defined here and the fact that the agents will never complete a trade that is not strictly utility-improving for both individuals. See Table 10 below for a summary of the average distance from predicted equilibrium holdings across both individuals in each round as well as means across the ten rounds for each treatment.

Table 10. Proximity to equilibrium (in assets)

		treatment		
		negative	zero	positive
round	1	5.5	1	4.5
	2	3	1	4.5
	3	5.5	1	5.5
	4	3	1	5.5
	5	4.5	0.5	4.5
	6	3	1	4.5
	7	5.5	0.5	4.5
	8	3	0.5	4.5
	9	4.5	0.5	5.5

10	5.5	0.5	4.5
means	4.3	0.75	4.8

With this metric, too, the zero treatment seems to argue against the initial hypothesis of this research. While the result that the mean of the positive treatment (4.8) is greater than the mean of the negative treatment (4.3) is mildly supportive of our hypothesis, it is interesting and not immediately apparent why the mean for the zero treatment is not only smaller than that of the negative treatment at 0.75 but substantially so.

The last formal measure of efficiency this research will discuss is another form of proximity to equilibrium, this time in terms of mean-variance utility rather than holdings. The logic, however, is the same in that markets that get closer to their predicted equilibriums are more allocationally efficient than those that do not get as close. Predicted equilibrium utilities are calculated based on the predicted equilibrium holdings of the risky assets A and B described in the last metric. Predicted equilibrium holdings of cash are calculated by adding the net change in cash that would result if each transaction that an individual participated in to move their endowed holdings of the risky assets towards their desired equilibrium holdings occurred at equilibrium prices to their endowed cash holdings. These predicted equilibrium holdings are then translated into utility and compared against the individuals' actual ending utility based on whatever their actual ending holdings were at the end of a round. For each of the 30 rounds, the average distance from predicted equilibrium utility (in absolute value) across the two individuals is given. Note, however, that only one individual's (absolute value) distance from their predicted equilibrium actually needs to be considered to form this table. This is due to the zero-sum nature of the relationship between one individual's over- or underperformance and that of the other individual. Because the economy is closed and the fundamental preferences of the individuals are the same in all rounds (both individuals have the same utility function, risk aversion, etc.), any amount of overperformance of one individual must necessarily come at the expense of an underperformance of the exact same magnitude on behalf of the other individual (i.e. individual 1 overperforming by 5 utils necessitates that individual 2 underperforms by 5 utils). Means across the ten rounds of each treatment are given in the last row of the Table 11.

Table 11. Proximity to equilibrium in utilities (averaged across both individuals)

negative	zero	positive
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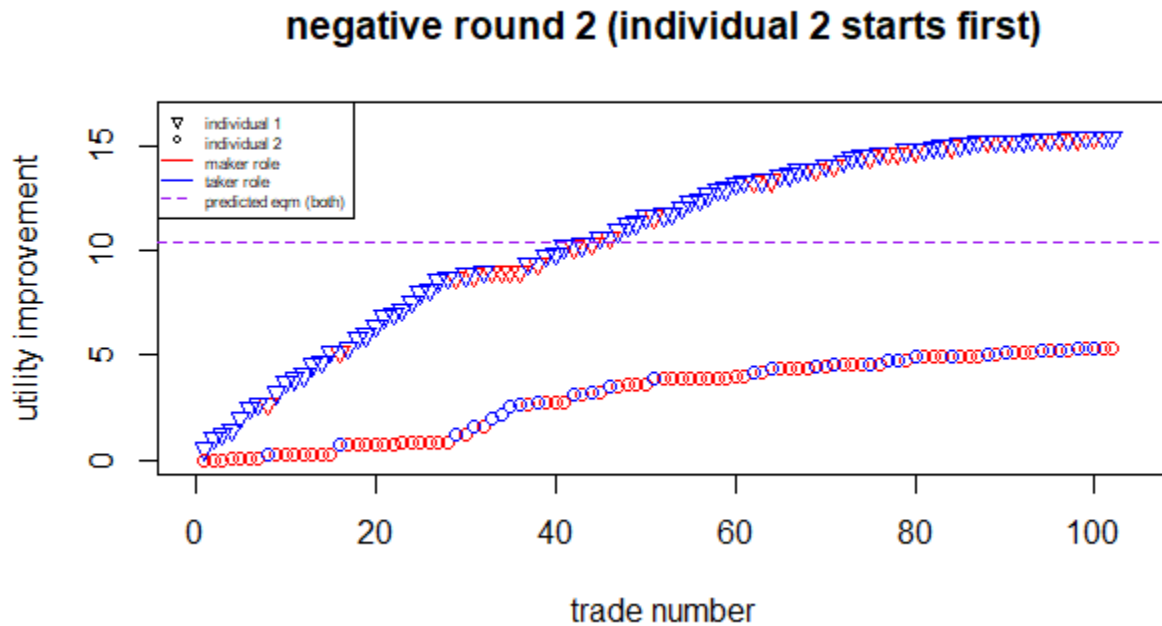
	1	5.76	0.36	2.6
	2	5.04	5.03	2.63
	3	6.74	3.16	2.44
	4	7.8	2.51	6.77
round	5	5.25	0.41	4.68
	6	4.79	2.79	4.93
	7	5.6	0.63	2.5
	8	6.72	1.07	2.49
	9	3.63	3.54	2.48
	10	0.2	4.48	1.33
	means	5.153	2.398	3.285

As evident from Table 11, proximity to equilibrium measured in terms of utilities does not support the initial hypothesis of this research. Individuals participating in markets consisting of negatively correlated assets are furthest on average from their predicted equilibrium with an average distance of 5.153. Positive rounds get closer, averaging only 3.285 utils away from predictions, while the zero rounds get closest, with an average distance of only 2.398. Conjecture about what may be causing this result is provided in the concluding discussion.

One last method to try and better understand some of the differences between the equilibration paths followed in the different treatments is to look at the utility improvements of each individual throughout a given round of trading. Ideally, a graph from the round that was ‘most efficient’ and a graph from the round that was ‘least efficient’ would be included for each treatment. This way, the graphs from the most efficient rounds and least efficient rounds could be compared against each other both within treatments and across treatments, in hopes of finding clues about what makes for an efficient round and what makes for an inefficient round and also what is unique to each treatment and may make it more or less conducive to efficient rounds. The minimum number of trades required to reach equilibrium in a negative treatment was 102, and round 2 is an example of one of the rounds that required this number of trades. Figure 5, shown below, tracks each individual’s utility improvement over the course of trading. The purple line indicates both individuals’ predicted utility improvement. From this, it can be seen that individual 1 overperforms in this round and individual 2 underperforms. Notice also that individual 1’s line includes more blue points than red, this is also in line with expectations, as assuming the ‘taker role’ in the majority of transactions is likely to cause an individual to outperform in that round. Notice also the almost flat slope surrounding the red points in both

individuals' utility improvement lines, which again shows how minorly beneficial it is for an individual to complete a transaction in which they have assumed the 'maker role'. Of course, this would not be the case if the trading agents were given larger spread parameters.

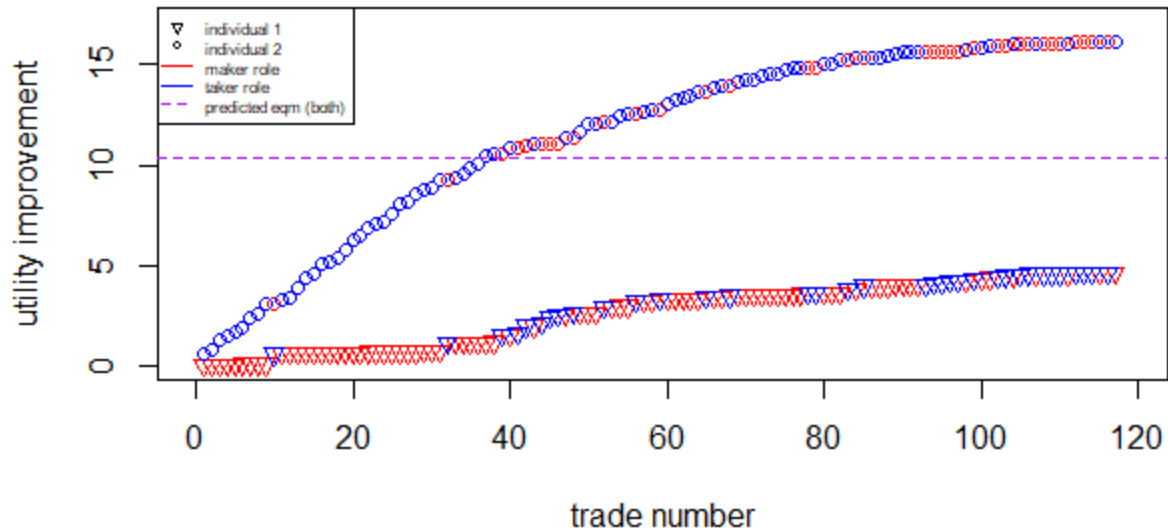
Figure 5. Tracking both individual's utility improvements over the course of trading in the second round of the negative treatment.



The maximum number of trades required to reach equilibrium in the negative treatment was 117. The first round took this number of trades, making it a good example of a less efficient negative treatment round. Figure 6, provided below, shows utility improvements for this round.

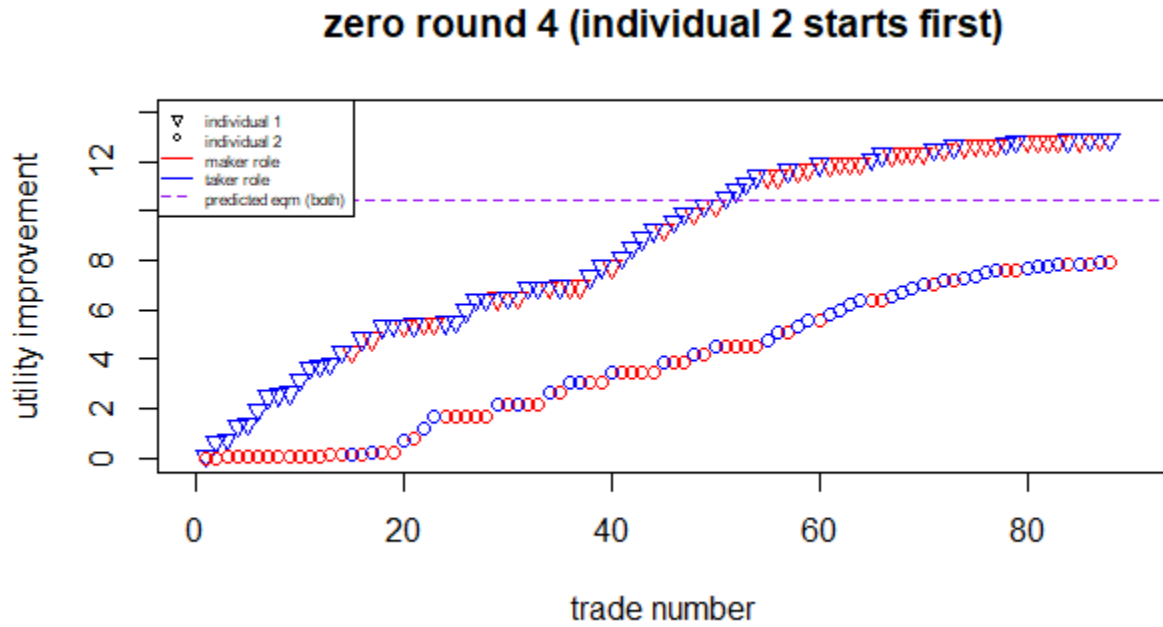
Figure 6. Tracking both individual's utility improvements over the course of trading in the first round of the negative treatment.

negative round 1 (individual 1 starts first)



Unfortunately, the zero and negative treatments do not indicate much evidence of dispersion as far as some rounds within each treatment being more efficient and some being less. Because of this, instead of including graphs for two rounds (one efficient and one inefficient) from each, we choose the round that seems most ‘average’ to include for each treatment. We decide which round is most ‘average’ in each treatment by choosing the round which is closest to the average in the ‘absolute value of proximity to equilibrium utility’ metric. Numbers for this metric are included in Table 11. For the zero treatment, the round that appears most average by this method is round 4. Figure 7 below tracks the utility of both individuals over the course of trading in this round.

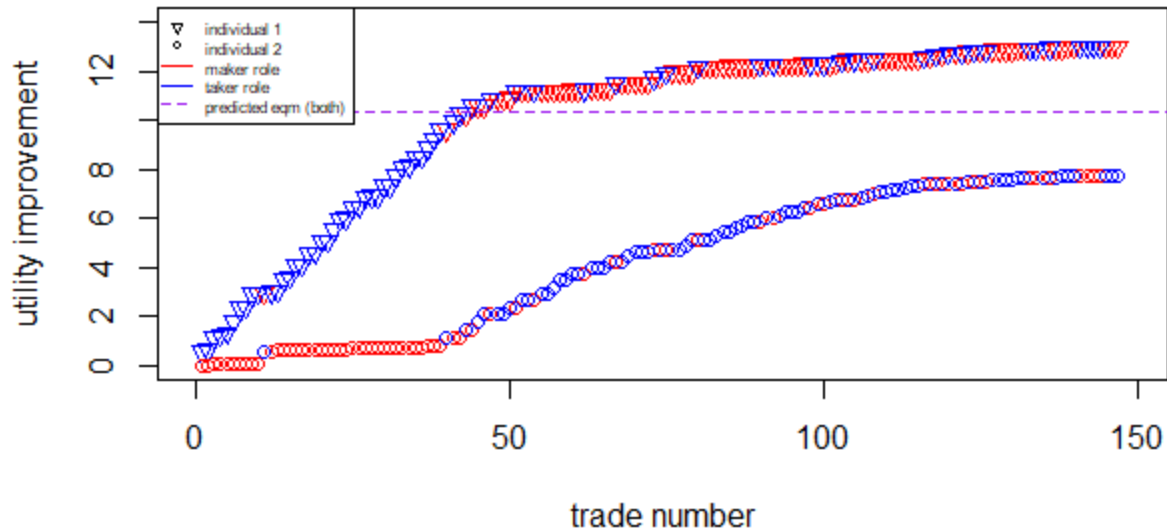
Figure 7. Tracking both individual's utility improvements over the course of trading in the fourth round of the zero treatment.



Lastly, round 2 of the positive treatment appears to be the most average out of the ten rounds. Figure 8, below, tracks the utility of both individuals over the course of trading in this round.

Figure 8. Tracking both individual's utility improvements over the course of trading in the second round of the positive treatment.

positive round 2 (individual 2 starts first)



While these graphs help to confirm intuition regarding the impacts of assuming the ‘maker’ versus ‘taker’ role in transactions, it is still hard to discern from them what role each treatment plays in causing more or less efficient trading rounds.

CONCLUDING DISCUSSION

When thinking about the results of this research as a whole it is useful to make a distinction between the efficiency of a market’s equilibration path and that of its actual equilibrium. Most of the measures included in this research that examine the efficiency of the equilibration path are supportive of the main hypothesis of this research – that markets consisting of negatively correlated assets are more allocationally efficient. The other measures included, however, those focusing on the efficiency of the equilibrium reached by a market (mostly related to the proximity of a given market’s equilibrium to its predicted equilibrium), are not supportive of the hypothesis and are instead, to some degree, directly contradictory. We believe this may be a result of the fact that the variance of asset B varies across the three treatments. In the positive and negative treatment, the variance of B is only $\frac{1}{6}$, while in the zero treatment it is $\frac{1}{2}$. Due to the mean-variance nature of the agents used in simulations, transacting in an asset with a larger variance means larger jumps in utilities and marginal valuations from one transaction to the next.

As a result, two individuals trading in an asset with a large variance would theoretically get much closer to their predicted equilibrium holdings before getting stuck by the fact that their valuations are within one cent of each other. The same two individuals trading in an asset with a small variance, however, may reach the point of their valuations being within one cent of each other's when they still have a much larger number of trades left before they reach predicted equilibrium holdings. This is because the smaller variance of the asset means that each individual is effectively less hurt (via decreased utility) from each unit away that they are from predicted equilibrium, and this diminished consequence is reflected in the increased speed with which their valuations converge on the path towards equilibrium holdings. To address these ideas, we would like to complete more simulations in the future using different asset structures so that while the correlation between assets in each treatment are the same as those in this research, the variances of the assets are held constant across treatments.

Another result of this research that is not fully understood is why which individual that underperforms and which that outperforms their expected equilibrium utility does not switch according to which individual starts first in a given round in the zero and positive correlation treatments. See Appendix A for a table showing which individual outperforms expected utility and which underperforms in each round and by how much. Asparouhova et al. (2020) explains how in a setting such as that of this research, where the market consists only of two makers both using small spreads, assuming the role of maker in any given transaction reduces the increase in utility that an individual experiences from the transaction. Note from the description of the experimental design of this research that in each round of trading, the individual that entered the market first was always deliberate. It would seem logical that this individual, due to their time (dis)advantage, would be likely to more often assume the role of maker in transactions for that round, simply as a result of their entering the market first. It would also be expected, then, that the individual who enters the market first would be the one to underperform their predicted equilibrium utility in that round. This is not what we observe. Instead, the negative treatment is the only of the three that comes close to following this pattern (the only round in which it does not is round 10, where individual 2 enters the market first and also outperforms their predicted equilibrium utility). In the zero treatment, individual 1 is the 'winner' at the expense of individual 2 in every round except 9, even though which robot starts first alternates exactly the same (individual 1 in odd rounds and 2 in even). The positive treatment violates the pattern even

more completely, with individual 1 gaining the greater utility improvement in all 10 rounds, again in spite of the fact that the starting robot is alternating through the 10 rounds.

The graphs included at the end of the Results section are helpful in thinking further about what might be going on. It is easy to understand that the key to being the ‘winner’ of any given round is assuming the role of taker in the majority of transactions. It is also easy to understand why starting first would make an individual less likely to be able to do this. However, starting first in our experimental setup does not necessitate occupying the ‘maker role’ for the entirety of the round. This is because there is some randomness in the order in which robots are able to get orders onto the market introduced by the issue of speed. While a type 1 individual may start first in a round, at some point in the round the type 2 individual may, for a variety of reasons primarily related to technical aspects of the functioning of the robots and their interaction with Flex-E-Markets such as server speed, etc., be faster to get a new set of orders on the market, and this will result in the type 2 individual assuming the role of maker for some number of transactions thereafter. This has interesting implications that do not necessarily agree with the seemingly intuitive results from work such as Baron, Brogaard, and Kirilenko (2012) that increased speed is beneficial. From this it can be understood why the robot that starts first will not, as a result of this randomness, be the one to lose out in the round 100% of the time. What is not understood, however, is why this randomness does not seem to affect the three treatments similarly. In the negative rounds, which robot starts first *is* almost a perfect predictor of which will lose out in the round. While in the zero and positive treatments, not only does which robot start first seem to have no power as a predictor of which will lose out, there is clearly something about individual 1 that makes it more well-equipped to outperform. What quality inherent to individual 1 that is missing in individual 2 and ultimately giving individual 1 the upper hand in these rounds is not known or understood. What can be concluded from this research, however, is that speed plays a large role in determining one’s profits relative to the individuals they compete against, the sign of the relationship between speed and profits is not always as expected, and the security design of the markets in which individuals participate seem to impact how important a role speed plays in determining relative profits.

It is clear that we do not yet understand all of the results we have seen. While we had always intended to run some algorithmic trading agent simulations as a proof of concept, the pandemic has limited us entirely to such simulations using MVO trading robots, and made it

impossible to conduct experimental sessions with human subjects. There have been silver linings to this forced change of plans, namely that we have come to much better understand the trading dynamics in such a robot vs. robot trading setup, and have observed interesting results related to speed, maker vs. taker roles, etc., such as those discussed earlier. This change has also, however, made it more difficult to fully answer the original questions of this research. The introduction of algorithmic agents has, at times, made it difficult to disentangle the results caused by correlation treatments from those that are simply a byproduct of the nature of the robots used. A fuller understanding of the questions posed by this research will likely have to wait until we are able to conduct experiments with human subjects.

APPENDIX A. UTILITY PROXIMITIES TO EQUILIBRIUM (BY INDIVIDUAL)

Table 12 shows each individual’s proximity to their predicted equilibrium utility in each round of each treatment. Note that the numbers reported in the table are a result of subtracting the actual ending utility of an individual from the predicted ending utility of the individual, meaning that a positive number is indicative of an individual falling short of their predicted utility in that round while a negative number is indicative of outperforming.

Table 12. Proximity to predicted equilibrium utility by individual

		treatment					
		negative		zero		positive	
		individual 1	individual 2	individual 1	individual 2	individual 1	individual 2
round	1	5.76	-5.76	-0.36	0.36	-2.6	2.6
	2	-5.04	5.04	-5.03	5.03	-2.63	2.63
	3	6.74	-6.74	-3.16	3.16	-2.44	2.44
	4	-7.8	7.8	-2.51	2.51	-6.77	6.77
	5	5.25	-5.25	-0.41	0.41	-4.68	4.68
	6	-4.79	4.79	-2.79	2.79	-4.93	4.93
	7	5.6	-5.6	-0.63	0.63	-2.5	2.5
	8	-6.72	6.72	-1.07	1.07	-2.49	2.49
	9	3.63	-3.63	3.54	-3.54	-2.48	2.48
	10	0.2	-0.2	-4.48	4.48	-1.33	1.33

Note the oddity in signs of values in the table. Intuition would suggest that for every odd-numbered round, individual 1 would have a positive value (indicating underperformance)

resulting from their starting first in the round and individual 2 a negative value (indicating overperformance) as a result of their starting second in the round. The opposite would be expected for even-numbered rounds, using similar logic. The only treatment in which this pattern holds is the negative treatment, where it holds in all rounds except round 10. In the zero treatment, which individual starts first seems to have very little impact on who overperforms and who underperforms. Instead, individual 1 outperforms in every round except round 9. The positive treatment upholds the pattern even less, with individual 1 outperforming in all ten rounds.

APPENDIX B. HOLDINGS OF ASSETS A IN ALL THREE TREATMENTS AND B IN THE ZERO TREATMENT

This research consists of three treatments for a two-asset market, meaning six total paths of equilibration to be examined (one for each asset in each treatment). The evolution of holdings along two of these paths (asset B in the negative and positive treatments) are not as expected. The paths followed in these assets/treatments and their oddities have been described in the results section. The holdings along the equilibration paths in the four remaining assets/treatments, in which trades do proceed as expected, are given here for reference.

Figure 9. Holdings of asset A in the negative treatment (with ‘trade number’ on x-axis including trades in A and B).

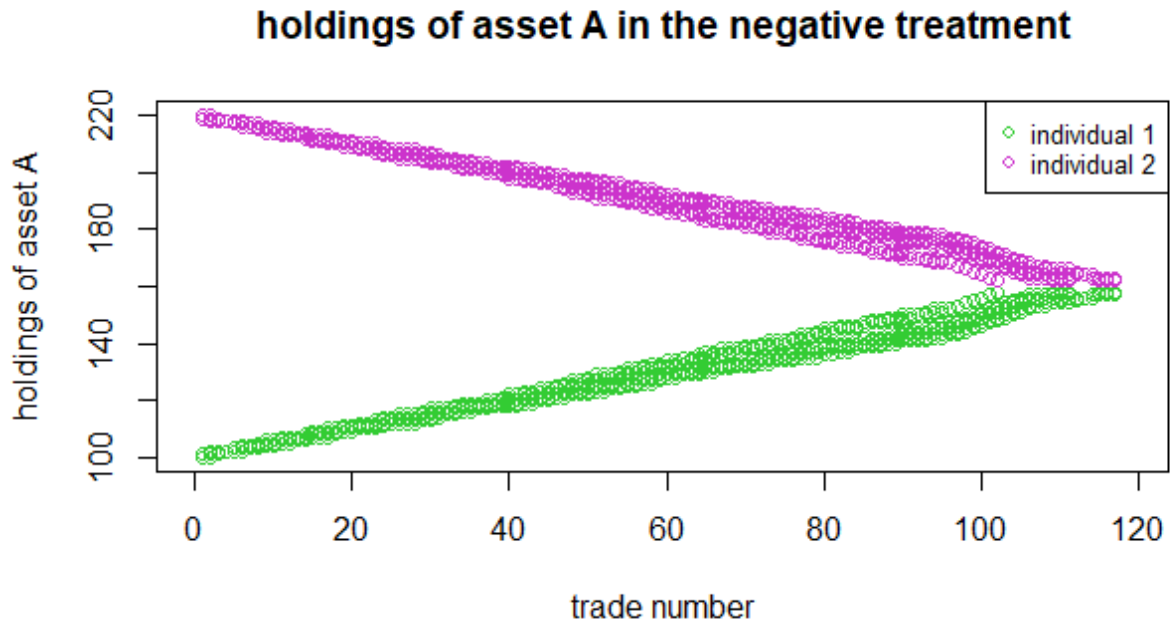


Figure 10. Holdings of asset A in the zero treatment (with ‘trade number’ on x-axis including trades in A and B).

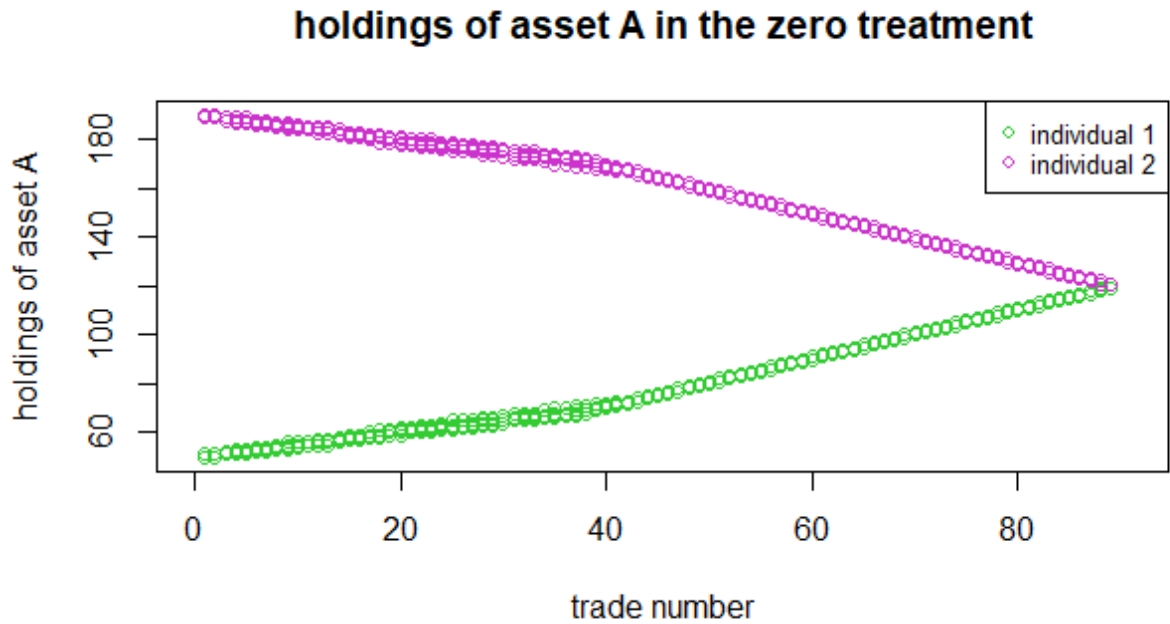


Figure 11. Holdings of asset A in the positive treatment (with ‘trade number’ on x-axis including trades in A and B).

holdings of asset A in the positive treatment

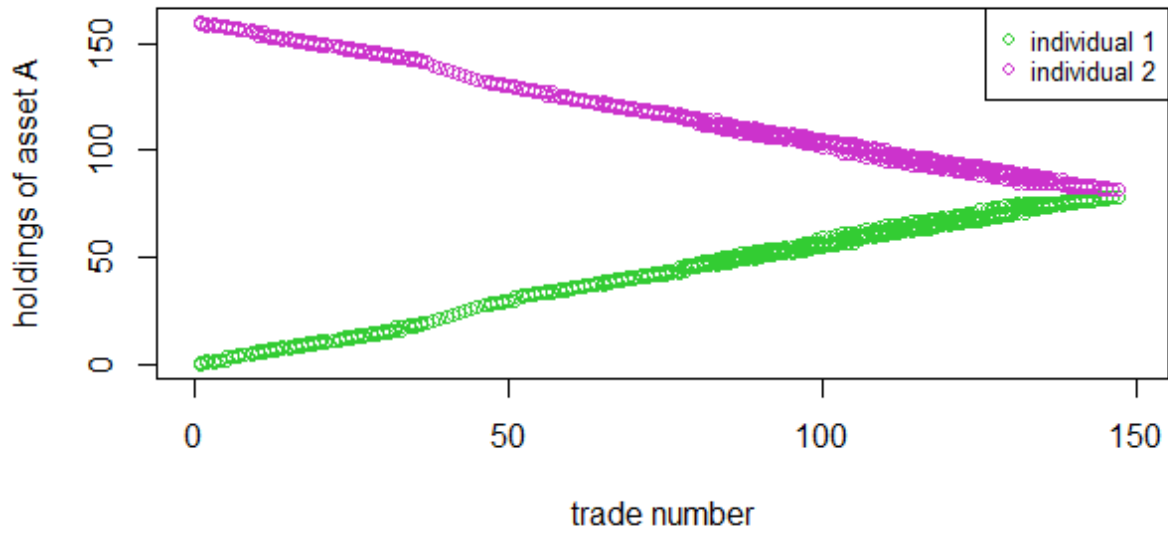
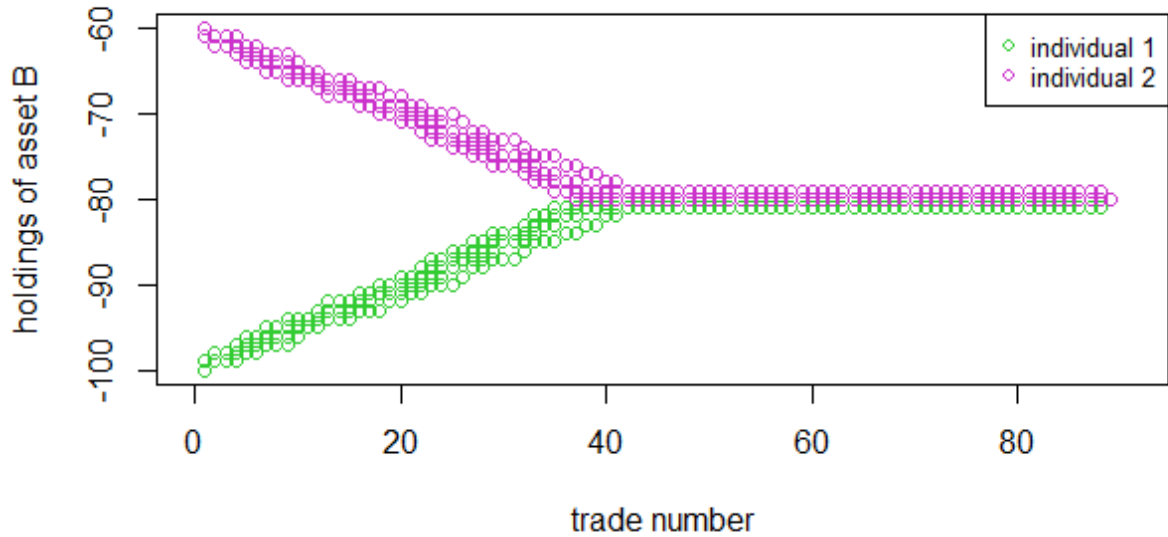


Figure 12. Holdings of asset B in the zero treatment (with ‘trade number’ on x-axis including trades in A and B).

holdings of asset B in the zero treatment



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