CHOPSAT: FEASIBLE REGION PROPERTIES

AMELIA LESSEN
ADVISOR: TOM HENDERSON

11 March 2023

Problem and Thesis

- The feasible region (F) is being studied to determine if it offers an efficient SAT solution or good heuristics for solving SAT

- The thesis of this work is that certain properties of F allow better insight into the existence of a SAT solution

- Due to the continuous nature of Hn, it may also be possible to estimate atom probabilities over all solutions

- The thesis examines linprog solutions, p-centers, analytic centers, and neural net models to evaluate the feasible region

The Satisfiability Problem

- The Satisfiability Problem (SAT) is to determine if there is a truth assignment to the logical variables that makes a logical sentence true
- If such a truth assignment exists, the sentence is called satisfiable
- Otherwise, the sentence is unsatisfiable
- The best known algorithm to solve SAT is NP-complete (meaning it requires polynomial time on a nondeterministic Turing machine)

Probabilistic Satisfiability

- Given n logical variables (or atoms) a model (or complete conjunction) is an assignment of 0 (false) or 1 (true) to each atom.
- There are 2^n models.
- These models can be represented as n-tuples in n-dimensional space, and correspond to the corners of H_n, the n-D hypercube.
- The meaning of this is that the ith axis corresponds to the values which can be assigned to the ith variable.
- Given any point in H_n, that point can be considered as a set of probabilities for the atoms.
- This allows consideration of a probabilistic version of SAT called Probabilistic SAT (PSAT)

Probabilistic Satisfiability

PSAT is defined as follows; given a logical sentence in Conjunctive Normal Form (CNF), and a probability, pi, associated with each conjunct, Ci, find a function, π : Ω → [0, 1], where Ω is the set of all complete conjunctions, and all of the following are true:

$$0 \leq \pi(\omega_k) \leq 1$$

$$\sum_{i=0}^{2^n-1} \pi(\omega_i) = 1$$

$$p_i = \sum_{\omega_k \models C_i} \pi(\omega_k)$$

The Satisfiability Problem

Henderson et al. proposed a geometric approach, called Chop-SAT for solving SAT

Some Explorations in SAT

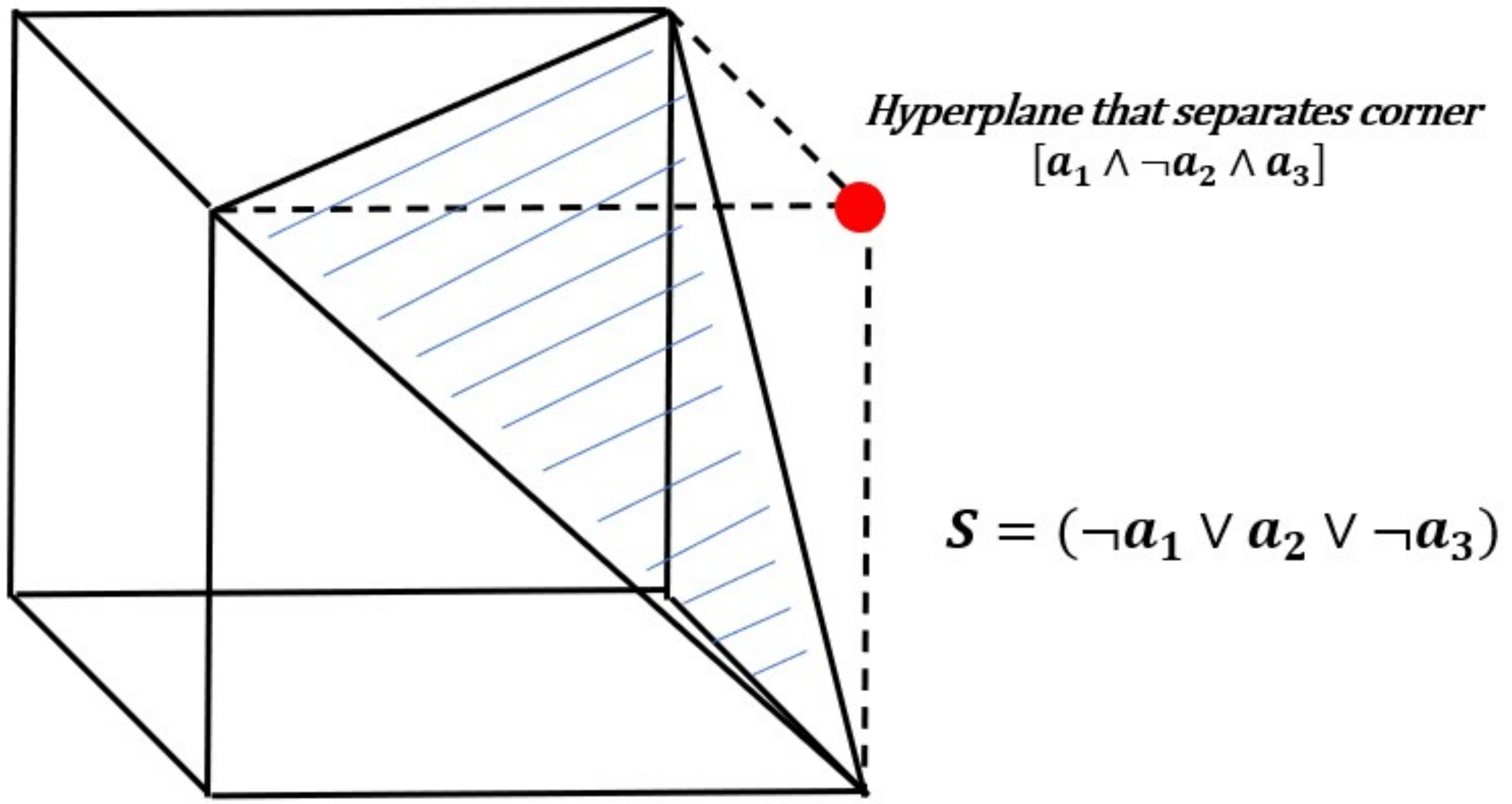
Thomas C. Henderson, Amar Mitiche, Xiuyi Fan, and David Sacharny
University of Utah

UUCS-21-016

School of Computing
University of Utah
Salt Lake City, UT 84112 USA

Geometric Approach to SAT

- In the geometric approach to SAT, the n-dimensional hypercube, H_n, represents the solution space for a SAT problem.
- Parts of H_n are removed based on a geometric interpretation of the logical sentence.
- This results in a convex feasible region, F, and a solution to the SAT problem is sought in F.

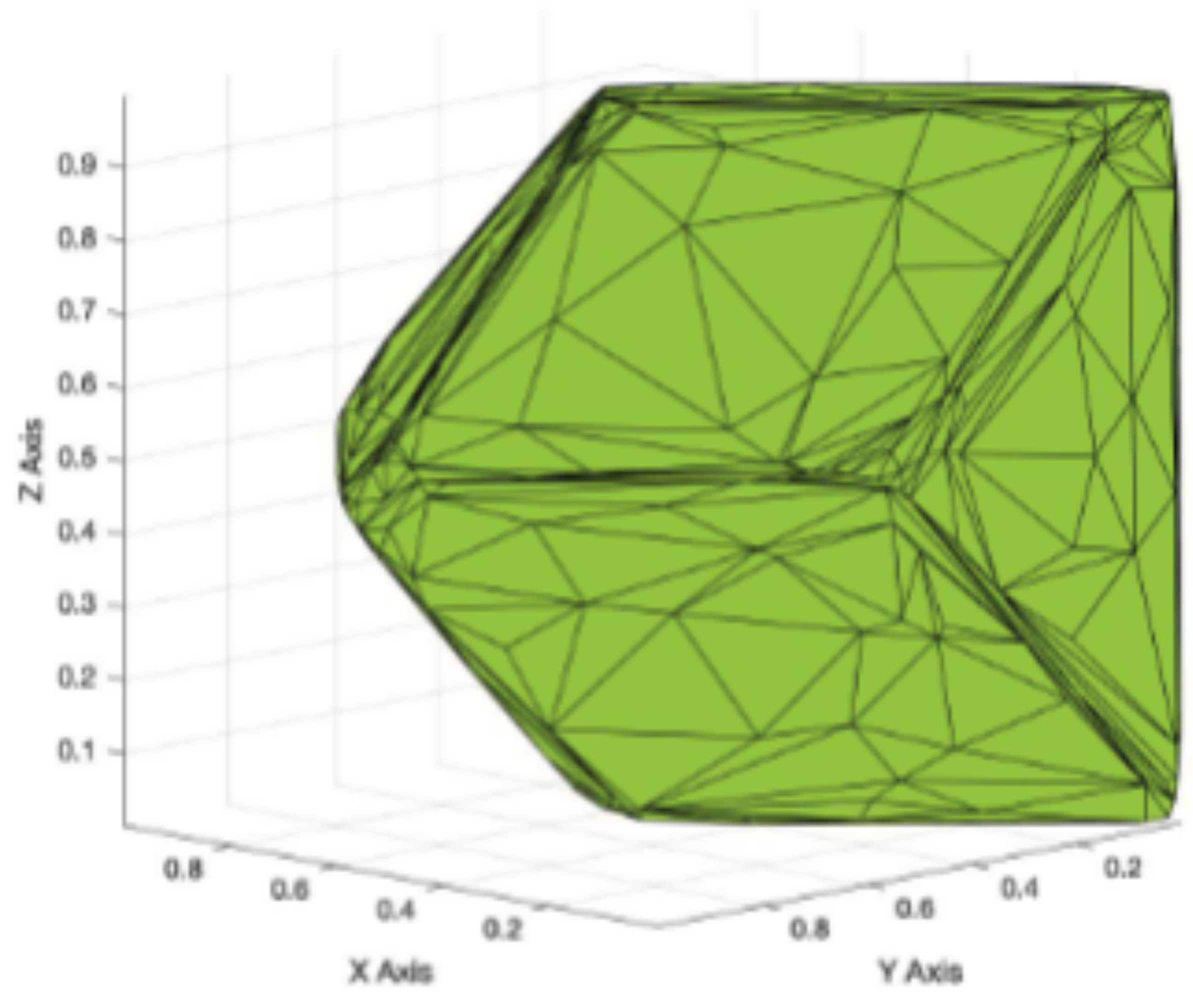


Chop-SAT: Feasible Region Properties

- A knowledge base (KB) is defined as a CNF sentence, and each conjunct is given a probability
- The Chop-SAT method is used to produce a set of hyperplanes such that the intersection of their non-negative half-spaces determines the feasible region
- The feasible region represents the solution space for the KB
- Every consistent sentence results in a feasible region which contains an H corner; thus, some point in the feasible region is $\sqrt{n}/2$ distant from the center of H, and it can be determined that the sentence is consistent

Chop-SAT: Satisfiable KB Example

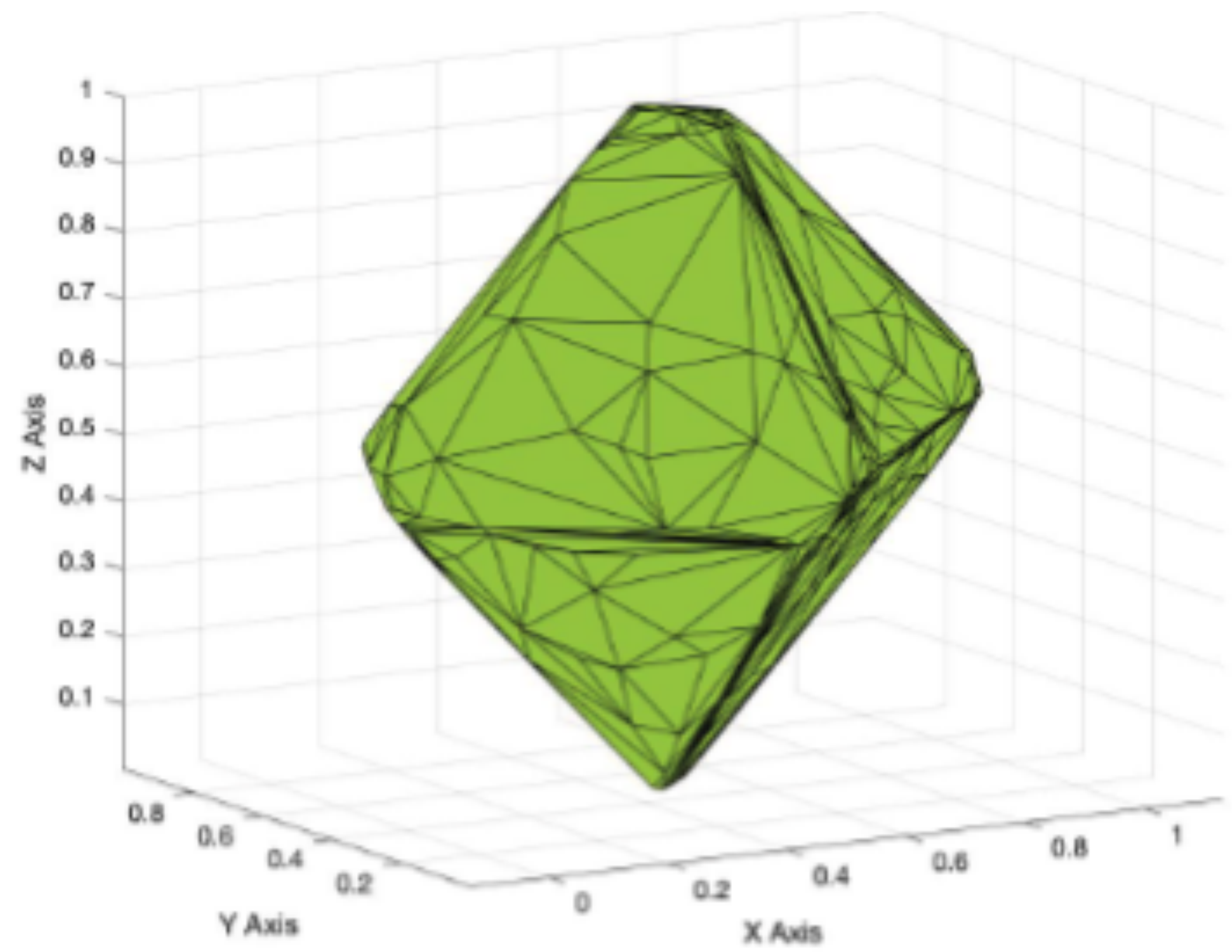
The clauses [-1 -2 -3], [-1 -2 3], [-1 2 -3], [-1 2 3], [1 -2 -3], and [1 -2 3], where each conjunct is represented by the index of the atom and a negative value if the atom is negated, form the feasible region below.



The feasible region for a satisfiable sentence in 3D.

Chop-SAT: Unsatisfiable KB Example

The clauses [-1 -2 -3], [-1 -2 3], [-1 2 -3], [-1 2 3], [1 -2 -3], and [1 -2 3], [1 -2 3], [1 2 3], where each conjunct is represented by the index of the atom and a negative value if the atom is negated, form the feasible region below.



The Feasible Region for the Unsatisfiable Sentence in 3D with maximal volume.

Linprog Solutions Method

- Examine usefulness of linprog solutions in determining the satisfiability of a knowledge base

How linprog works:

Linear programming solver

Finds the minimum of a problem specified by

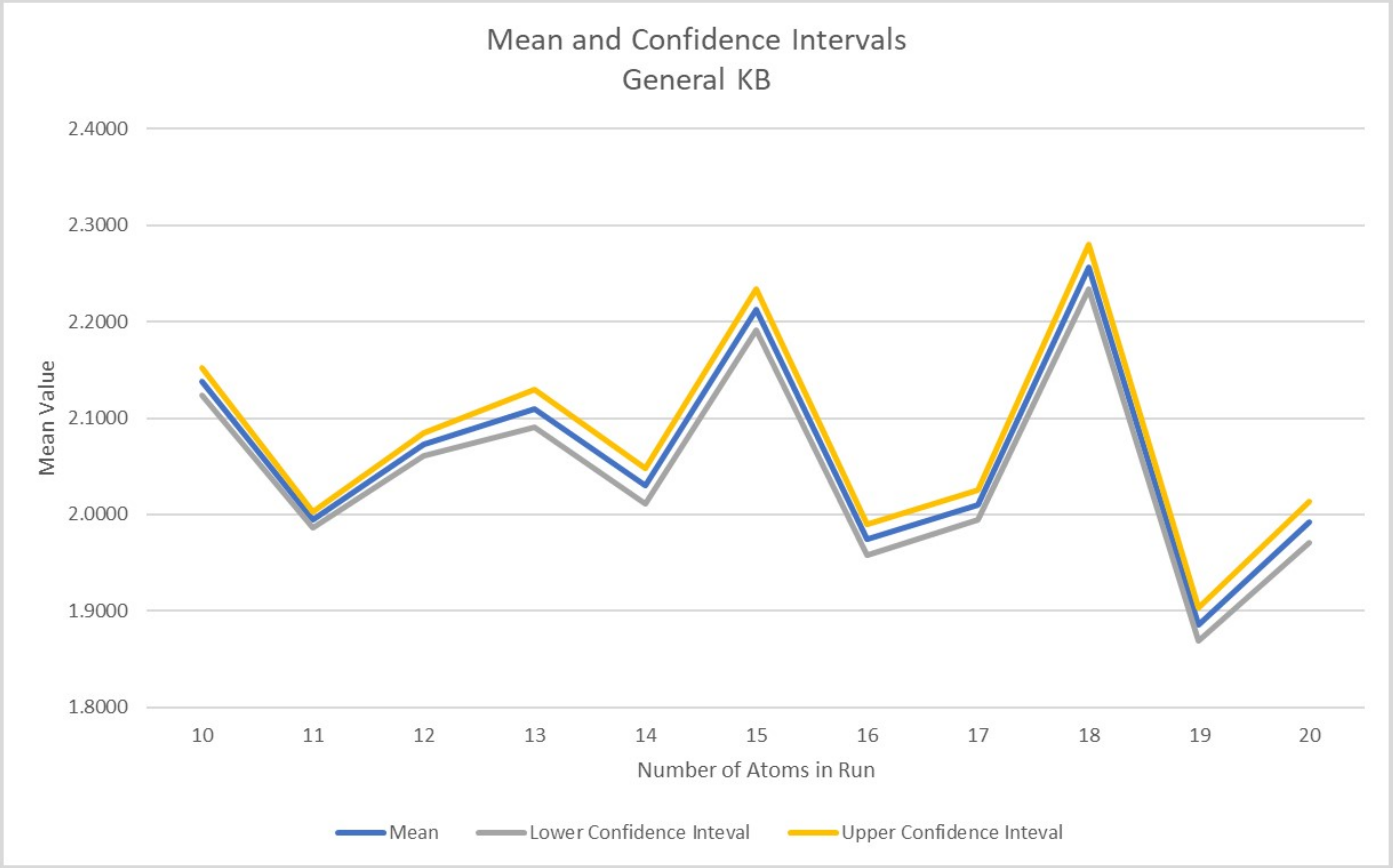
$$\min_{x} f^T x \text{ such that } \begin{cases} A \cdot x \leq b, \\ Aeq \cdot x = beq, \\ lb \leq x \leq ub. \end{cases}$$

f, x, b, beq, lb, and ub are vectors, and A and Aeq are matrices.

Linprog Solutions Method

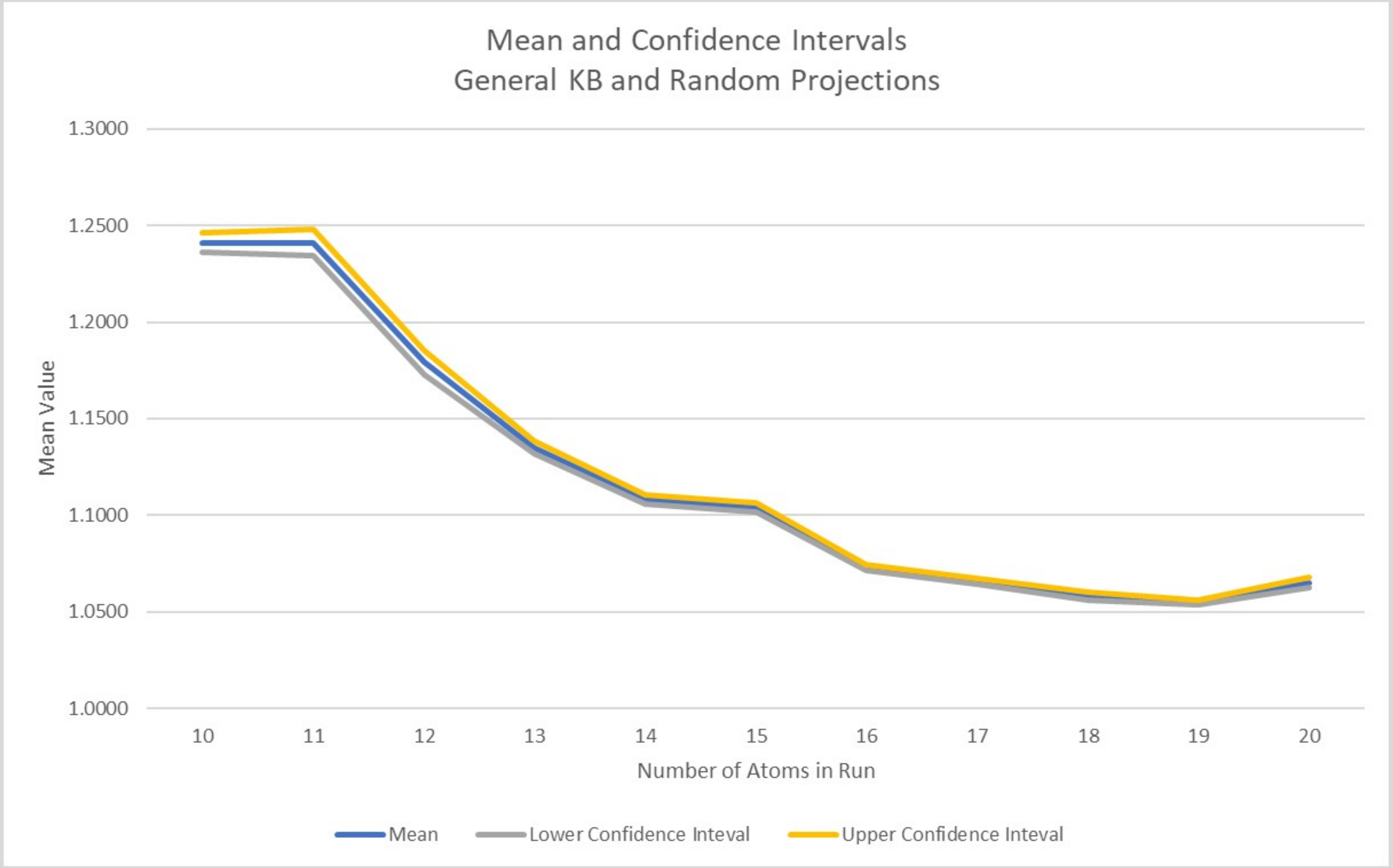
- The linprog solutions are determined for both general and independent knowledge bases with from 10 to 20 atoms
- In independent knowledge bases, atoms are guaranteed to be independent. That is, $P(a_1) * P(a_2) = P(a_1 \wedge a_2)$
- Declare knowledge base satisfiable when a linprog solution is found for which the distance from the center of a hypercube to the solution point found is greater than sqrt(n-2) / 2.
- In the maximal volume feasible region in an unsatisfiable KB, no point is farther than sqrt(n-2)/2 from the center of H_n

Linprog Solutions Method



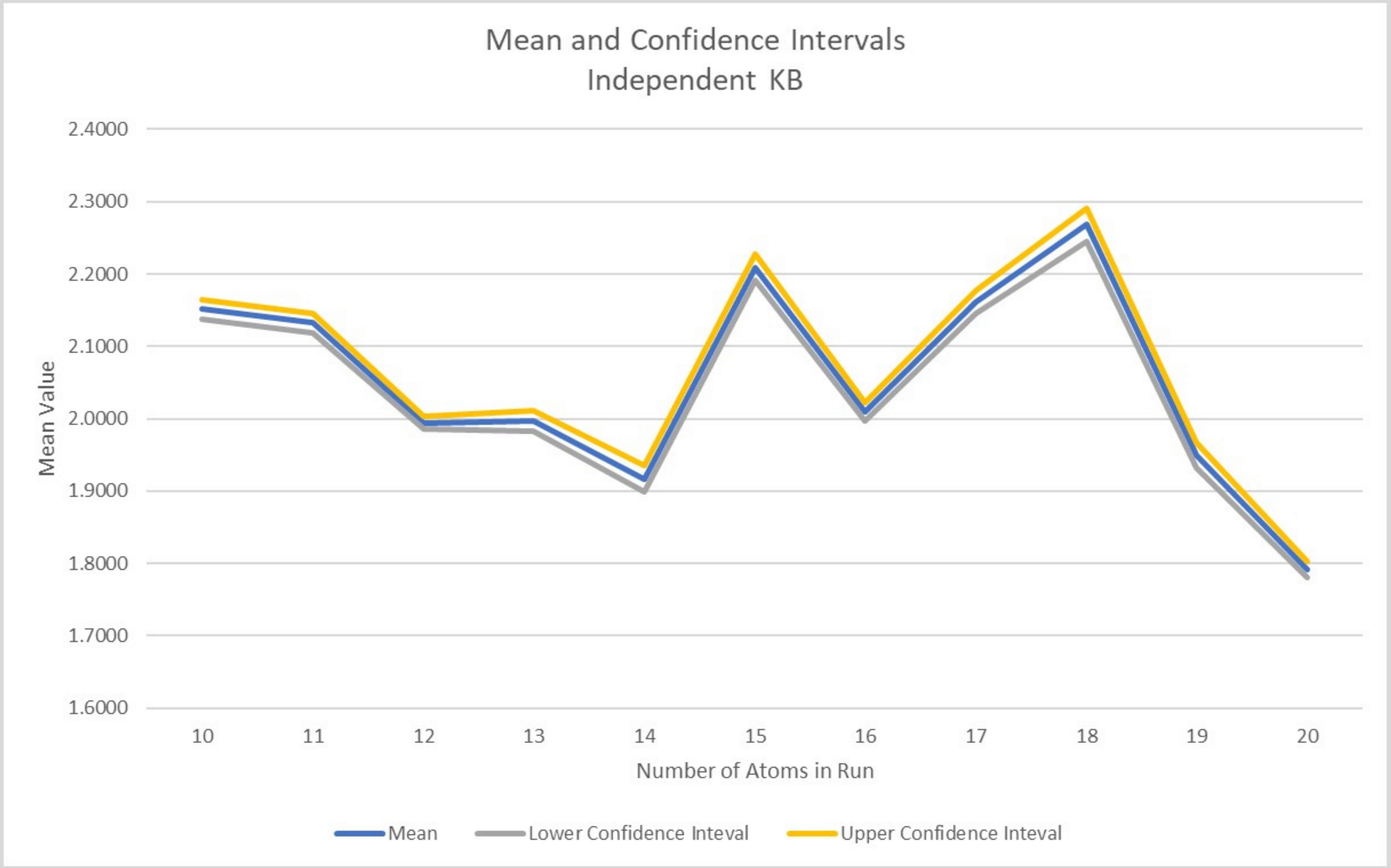
Plot of the average number of projections needed to determine a KB satisfiable on set axes on general KBs, from 10 to 20 atoms

Linprog Solutions Method



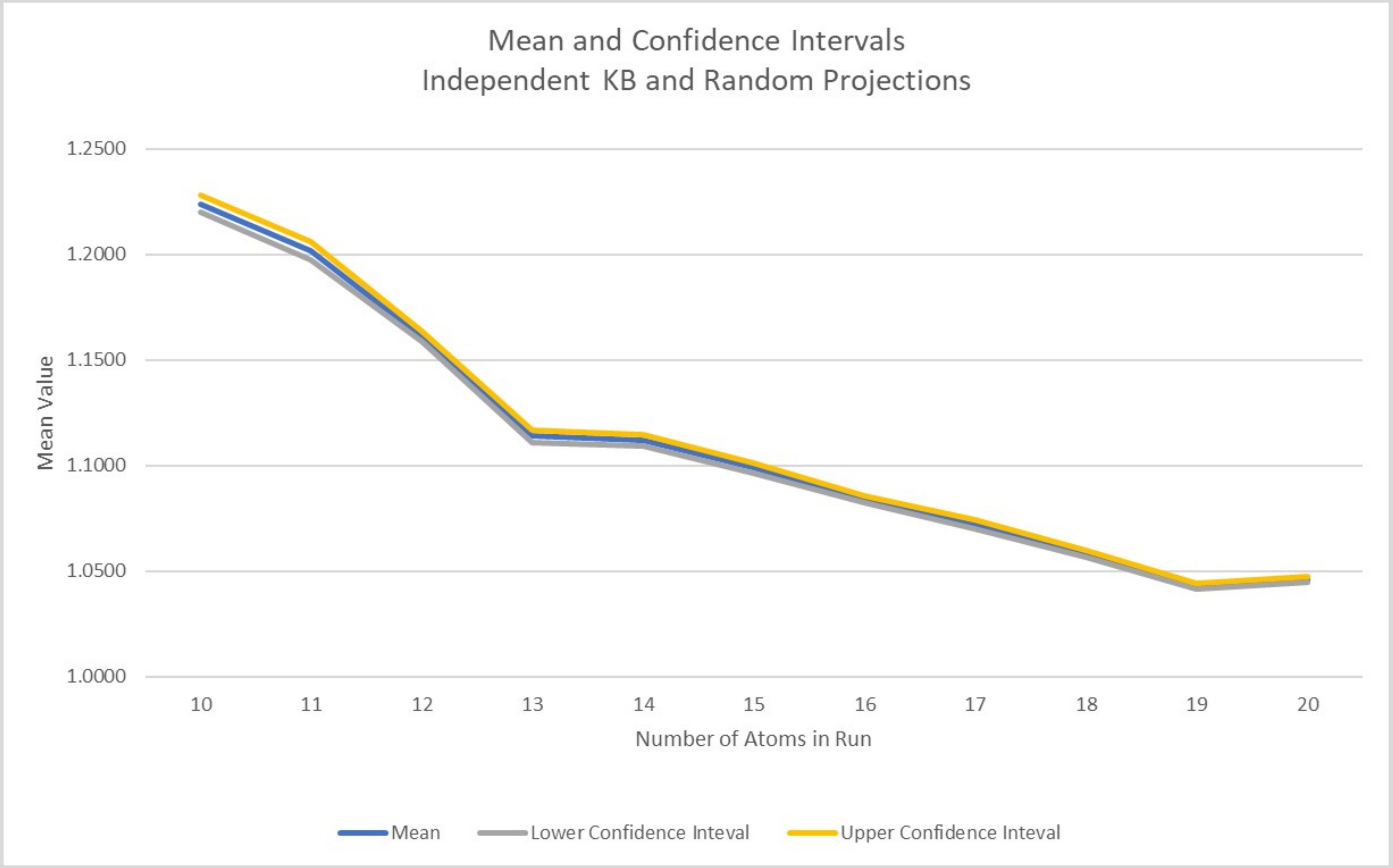
Plot of the average number of projections needed to determine a KB satisfiable on random axes on general KBs, from 10 to 20 atoms

Linprog Solutions Method



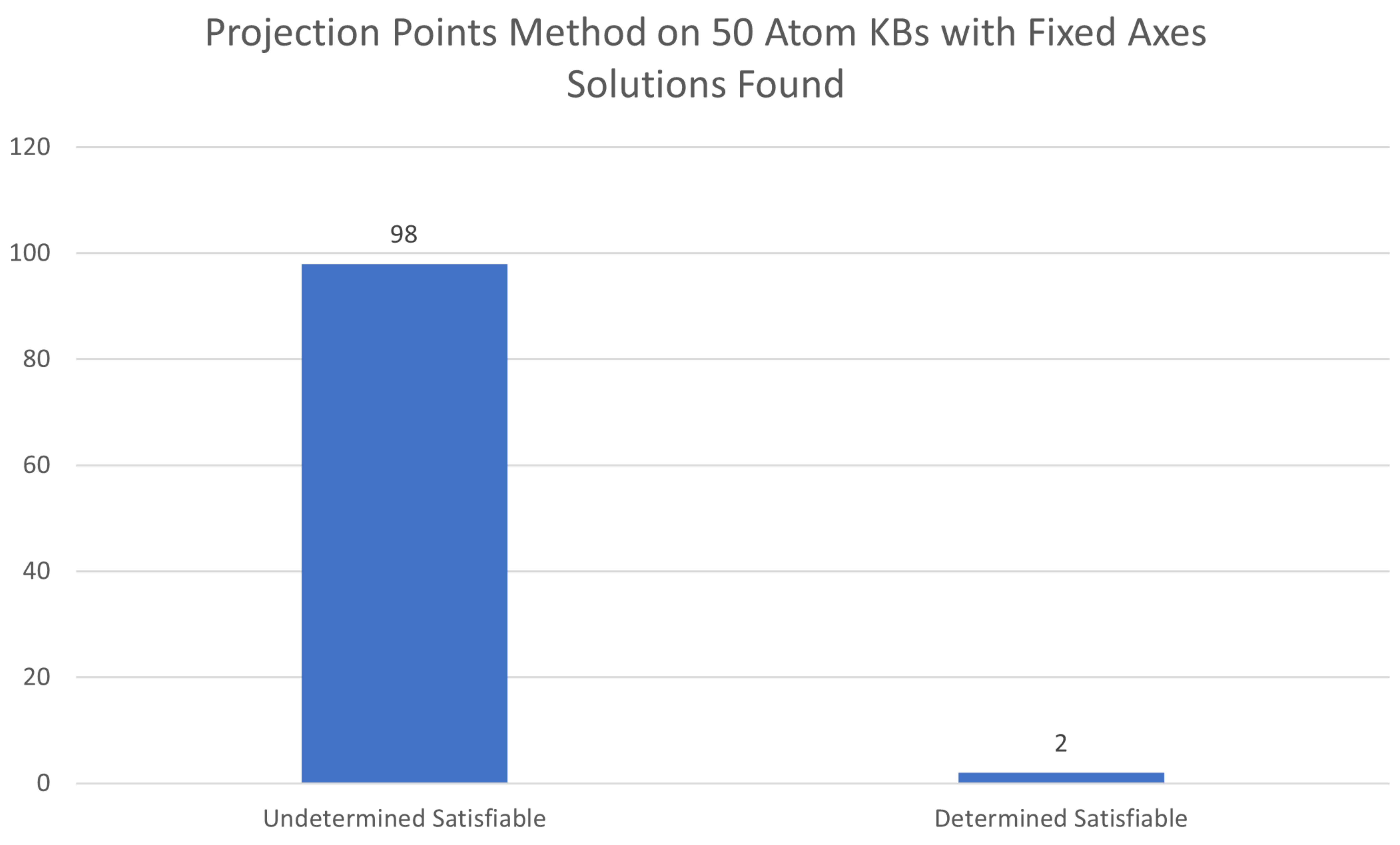
Plot of the average number of projections needed to determine a KB satisfiable on set axes on independent KBs, from 10 to 20 atoms

Linprog Solutions Method

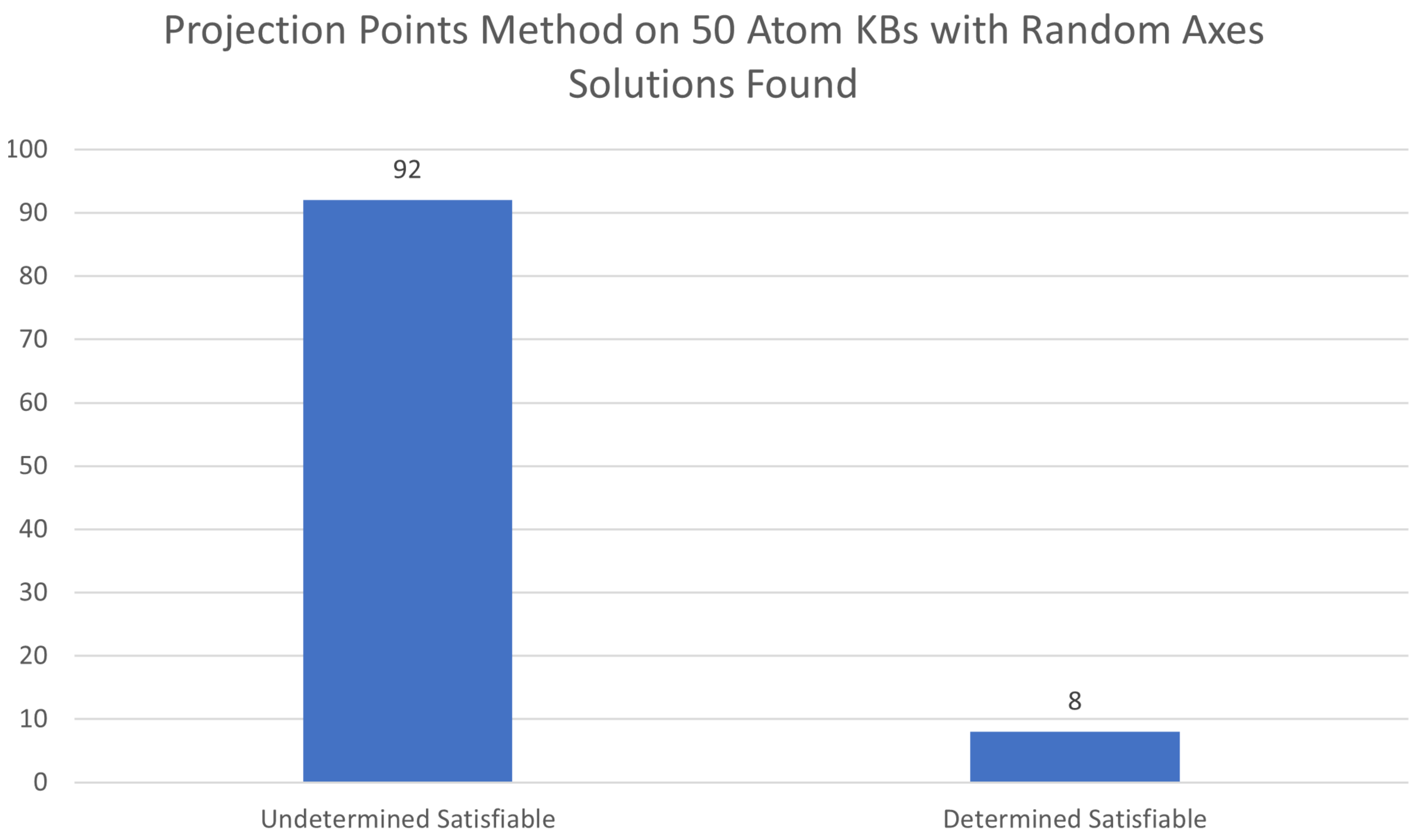


Plot of the average number of projections needed to determine a KB satisfiable on random axes on independent KBs, from 10 to 20 atoms

Linprog Solutions Method



Linprog Solutions Method



Linprog Solutions Method

- Method appeared promising on small KBs, but became time-consuming and performed poorly on medium-sized KBs
- Further testing on large KBs was not needed

Centers

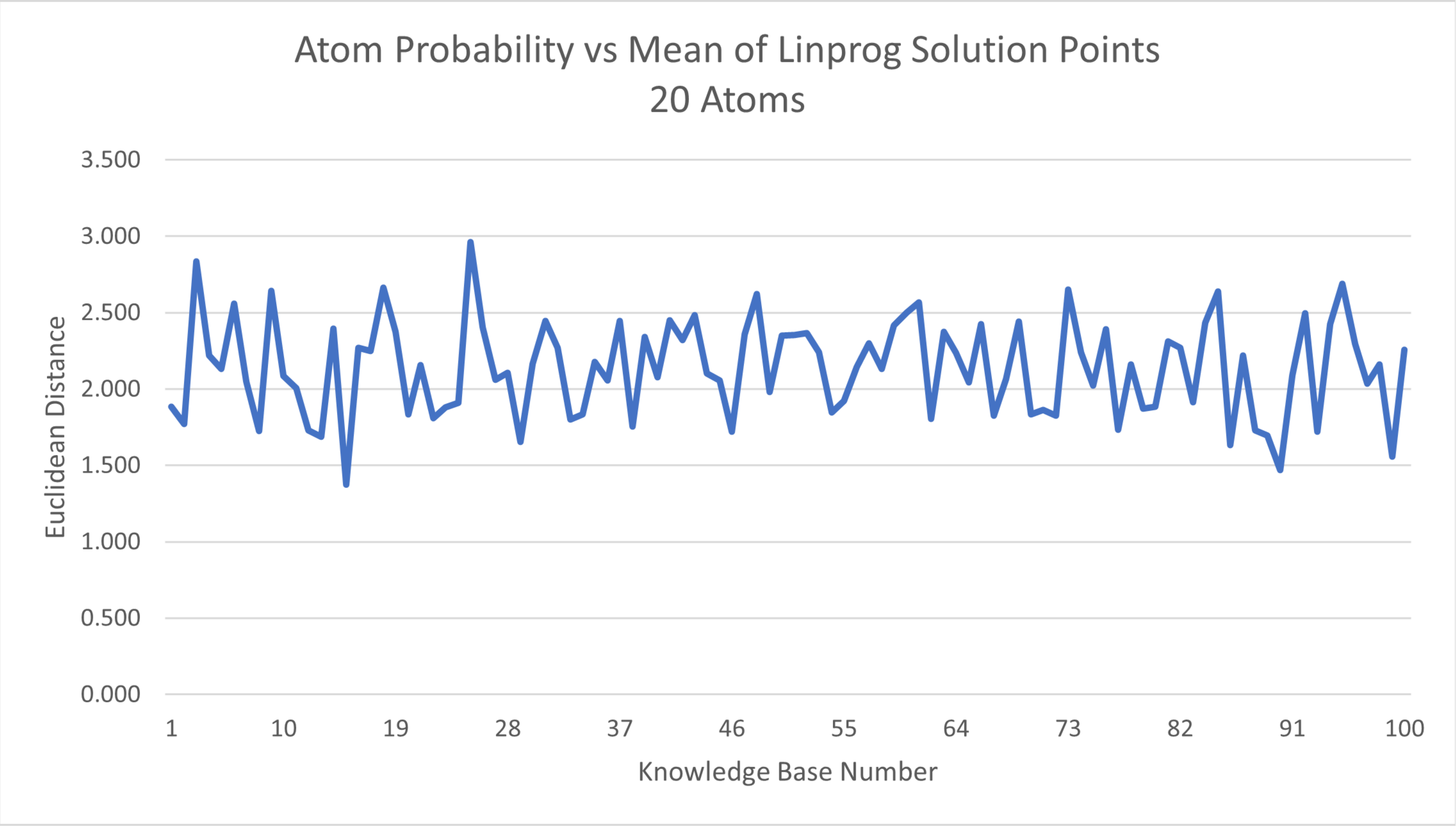
- In a KB with the clauses: $((a_1 \vee a_2 \vee a_3 \vee a_4) \wedge (\neg a_1 \vee \neg a_2) \wedge (\neg a_1 \vee \neg a_3) \wedge (\neg a_1 \vee \neg a_4) \wedge (\neg a_2 \vee \neg a_3) \wedge (\neg a_2 \vee \neg a_4) \wedge (\neg a_3 \vee \neg a_4))$, the solutions are: [0, 0, 0, 1], [0, 0, 1, 0], [0, 1, 0, 0], [1, 0, 0, 0].
- Each atom is true in 1/4 of the solutions, giving each atom a probability of 0.25.
- The probabilities of the atoms may be useful in a decision making process

Centers

Three center calculations to compare to atom probabilities

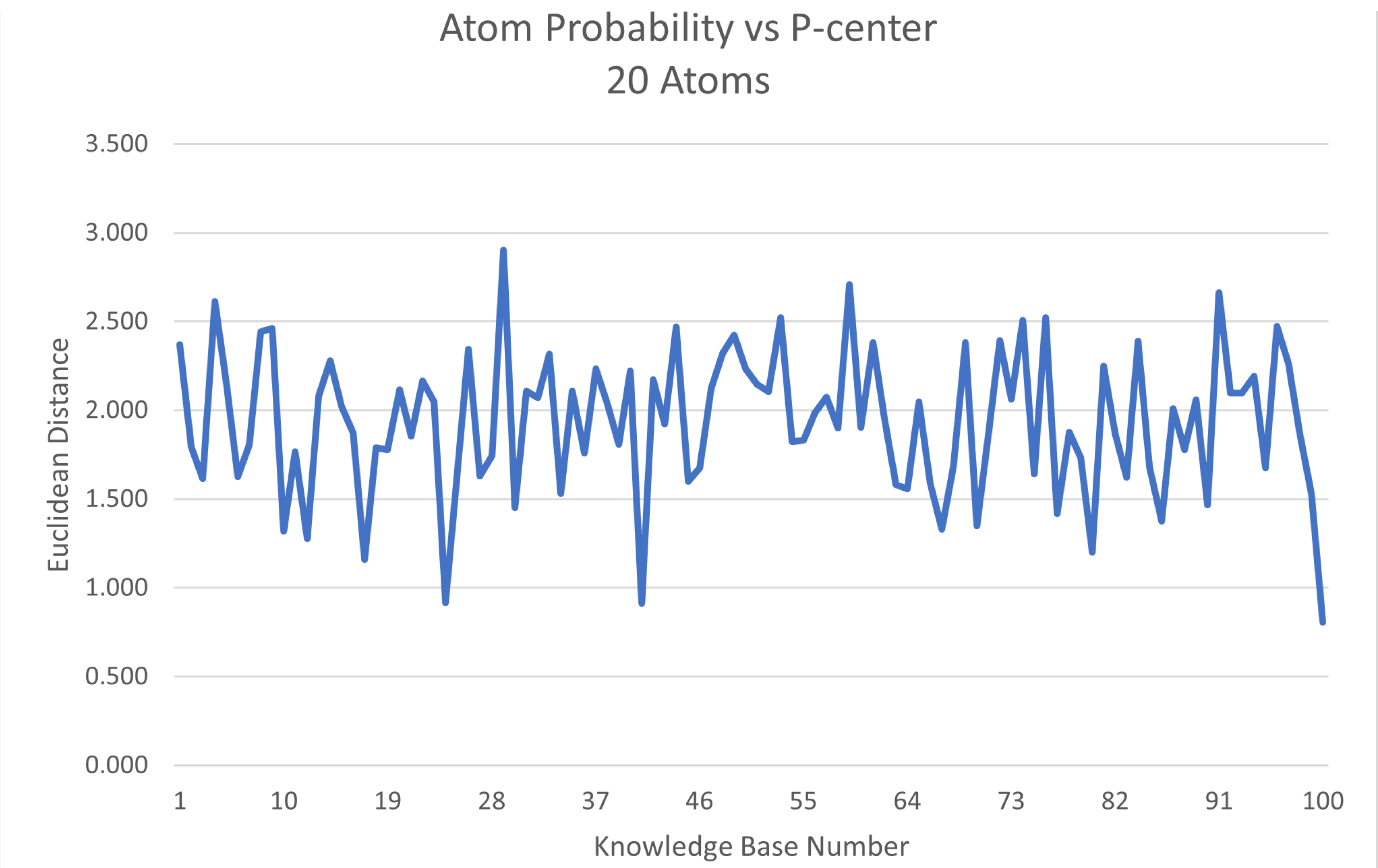
- Mean of linprog solutions
- P-center - point that minimizes the maximum distance between the hyperplanes
- Analytic Center - the y that maximizes $\prod \overline{a}^T \overline{y}$

Centers



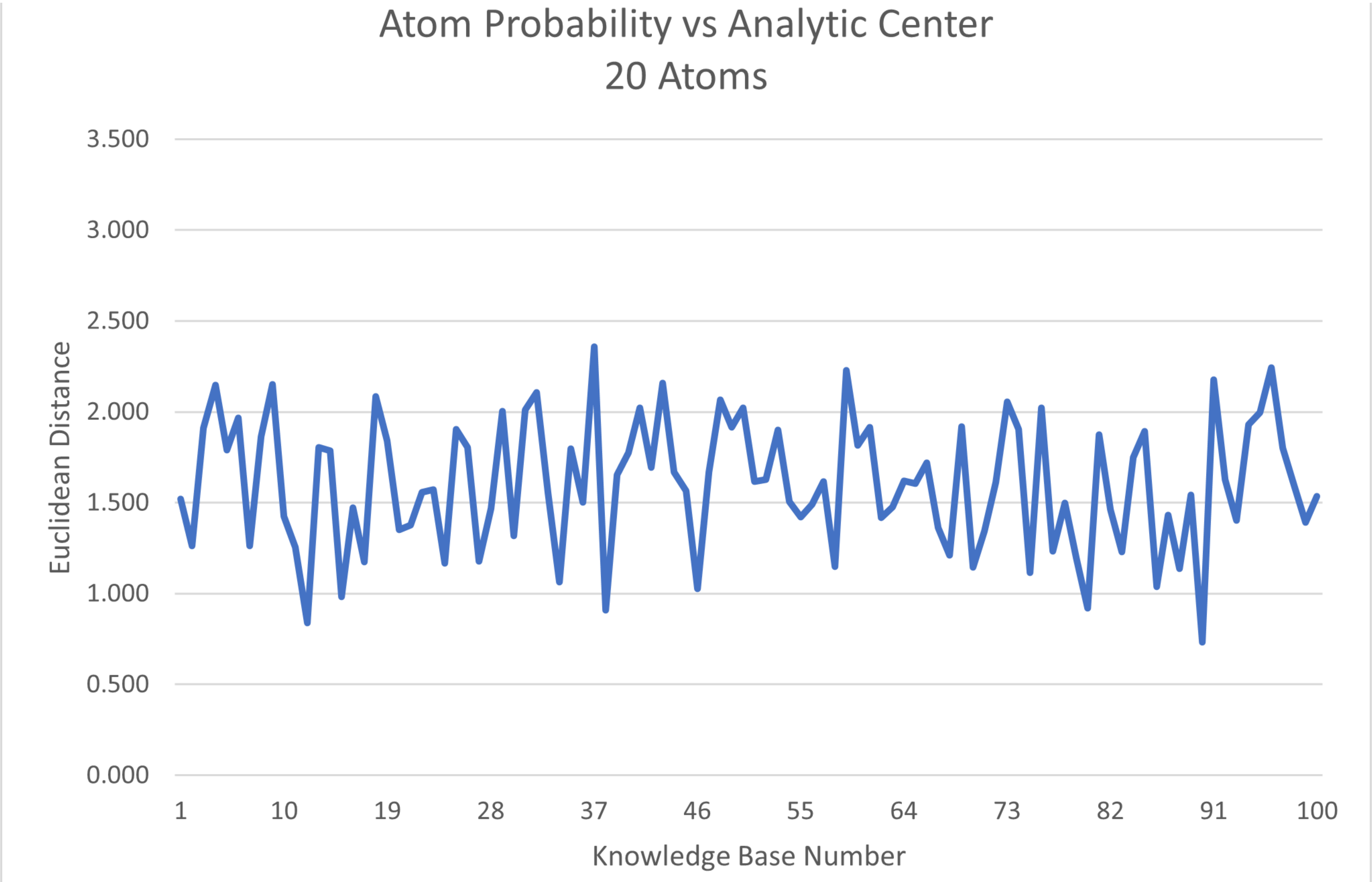
Plot of the Euclidean distance between the atom probabilities and the mean of the linprog solutions on satisfiable, 20 atom KBs.

Centers



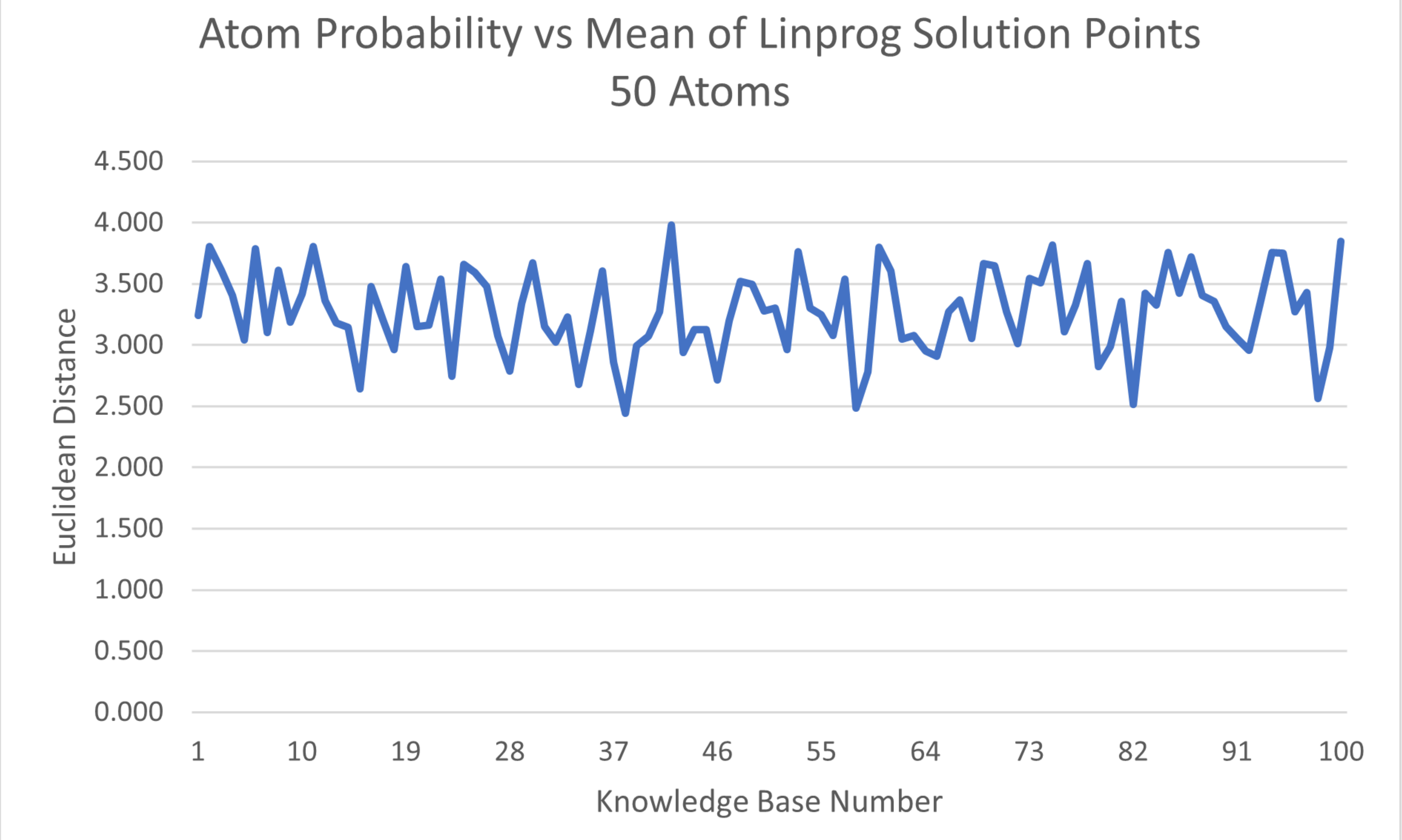
Plot of the Euclidean distance between the atom probabilities and the p-centers on satisfiable, 20 atom KBs.

Centers



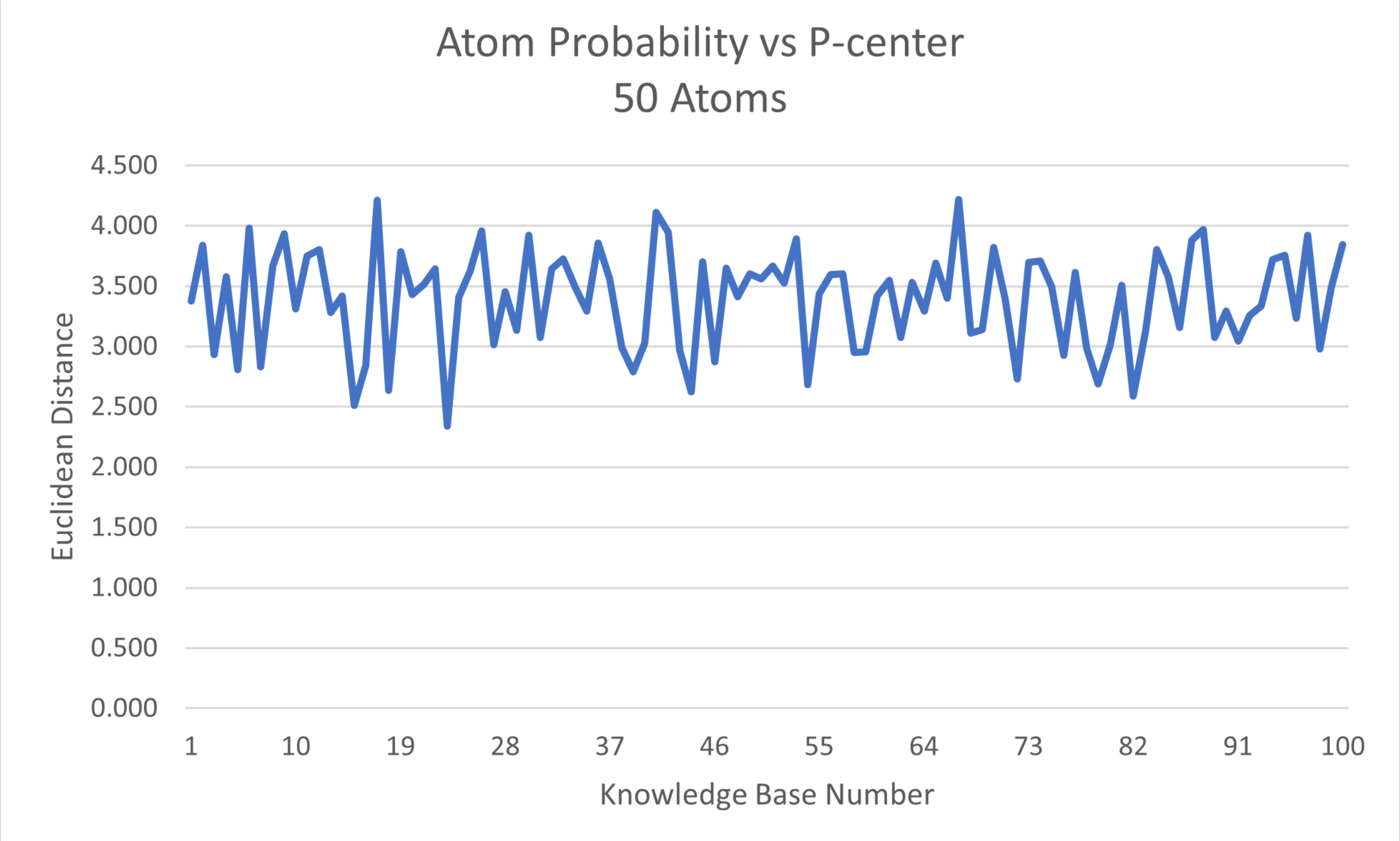
Plot of the Euclidean distance between the atom probabilities and the analytic centers on satisfiable, 20 atom KBs.

Centers



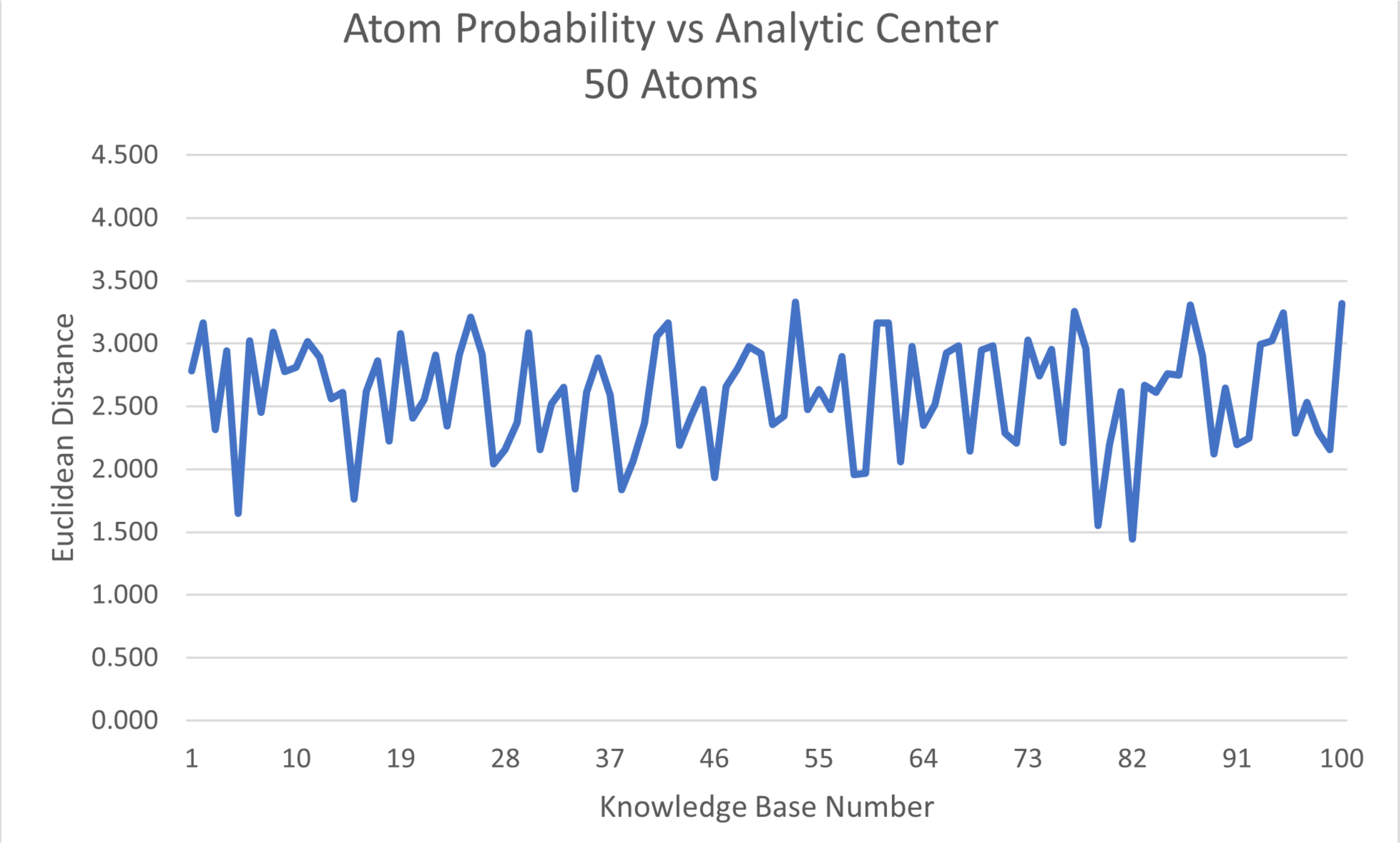
Plot of the Euclidean distance between the atom probabilities and the mean of the linprog solutions on satisfiable, 50 atom KBs.

Centers



Plot of the Euclidean distance between the atom probabilities and the p-centers on satisfiable, 50 atom KBs.

Centers



Plot of the Euclidean distance between the atom probabilities and the analytic centers on satisfiable, 50 atom KBs.

Neural Net Models

- Structure of neural net could be indicative of the satisfiability of the KB
- Train neural net models based on feasible points, unsatisfiable KB net and satisfiable KB net
- Create image of these neural net models
- Train neural net model with net images to classify KBs
- Trained 500 unsatisfiable KB neural nets and 500 satisfiable KB neural nets with 10000 points each
- Trained final neural net with images of above neural nets

Neural Net Models

- Classified 0.5050 of 200 Knowledge bases correctly
- Choosing satisfiable or unsatisfiable at random is almost equivalently as effective
- Structure of neural net is not indicative of the satisfiability

Conclusions

- Linprog solutions are not an accurate method for determining the satisfiability of a KB with more than a small number of atoms
- The analytic center is the best approximation to the atom probabilities
- The structure of a feasible point based neural net model does not indicate the satisfiability of the KB
- Further study would include using the analytic center in agent decision making situations to compare its effectiveness with other methods

References

[1] G. Georgakopoulos, D. Kavvadias, and C. Papadimitriou, “Probabilistic Satisfiability,” Journal Of Complexity, 4 (1987), pp. 1–11.

[2] Thomas C. Henderson, David Sacharny, Amar Mitiche, Xiuyi Fan, Amelia Lessen, Ishaan Rajan, and Tessa Nishida, “Chop-SAT: A New Approach to Solving SAT and Probabilistic SAT for Agent Knowledge Bases,” International Conference on Agents and Artificial Intelligence, Lisbon, Portugal, February, 2023.

[3] T. Henderson, A. Mitiche, X. Fan, and D. Sacharny, “Some Explorations In SAT,” Tech. Rep. UUCS-21-016, University Of Utah, Salt Lake City, UT, July 2021.

[4] N. Nilsson, “Probabilistic Logic,” Artificial Intelligence, 78 (1986), pp. 71–87.

[5] M. Sipser, Introduction To The Theory Of Computation, Cengage Learning, Independence, KY, 2012.

[6] Thomas C. Henderson, Amelia Lessen, Ishaan Rajan, and Tessa Nishida,“Chop-SAT: A New Method for Knowledge-Based Agent Decision making,” International Conference on Intelligent Autonomous Systems, Suwon, South Korea, July 2023, submitted.

Acknowledgements

This work was supported by funding from the Undergraduate Research Opportunities Program at the University of Utah awarded to Amelia Lessen.

Office of Undergraduate Studies
UNDERGRADUATE RESEARCH
THE UNIVERSITY OF UTAH

Acknowledgements

Professor Thomas Henderson